CS477 Formal Software Development Methods

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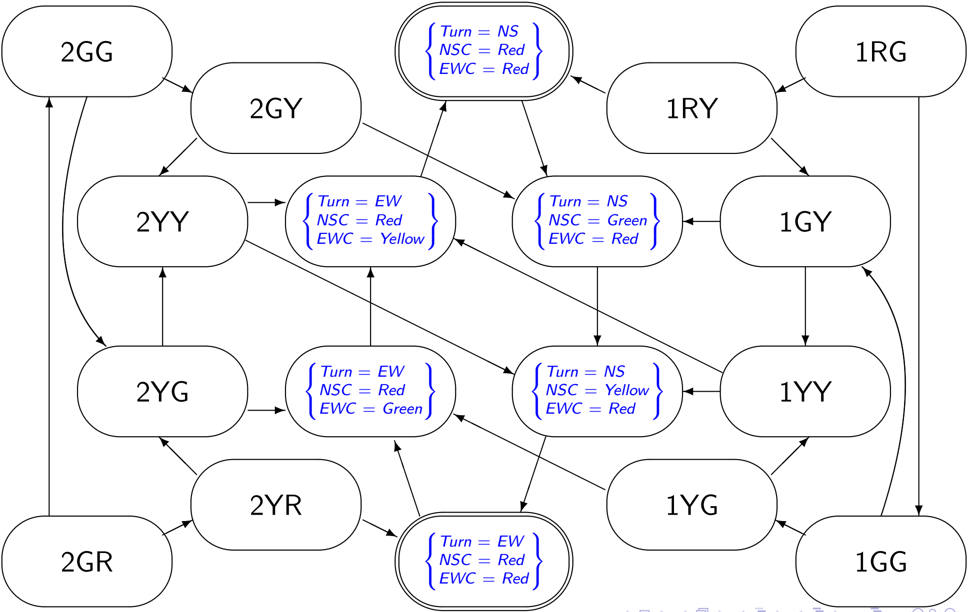
Example: Traffic Light

$V = \{Turn, NSC, EWC\}$, $F = \{NS, EW, Red, Yellow, Green\}$ (all arity 0), $R = \{=\}$

$$
\begin{array}{lrcl}
NSG & Turn = NS \wedge NSC = Red & \rightarrow & NSC := Green \\
NSY & NSC = Green & \rightarrow & NSC := Yellow \\
NSR & NSC = Yellow & \rightarrow & (Turn, NSC) := (EW, Red) \\
EWG & Turn = EW \wedge EWC = Red & \rightarrow & EWC := Green \\
EWY & EWC = Green & \rightarrow & EWC := Yellow \\
EWR & EWC = Yellow & \rightarrow & (Turn, EWC) := (NS, Red)
\end{array}
$$

$init = (NSC = Red \wedge EWC = Red \wedge (Turn = NS \vee Turn = EW)$

Example: Traffic Lights



Examples (cont)

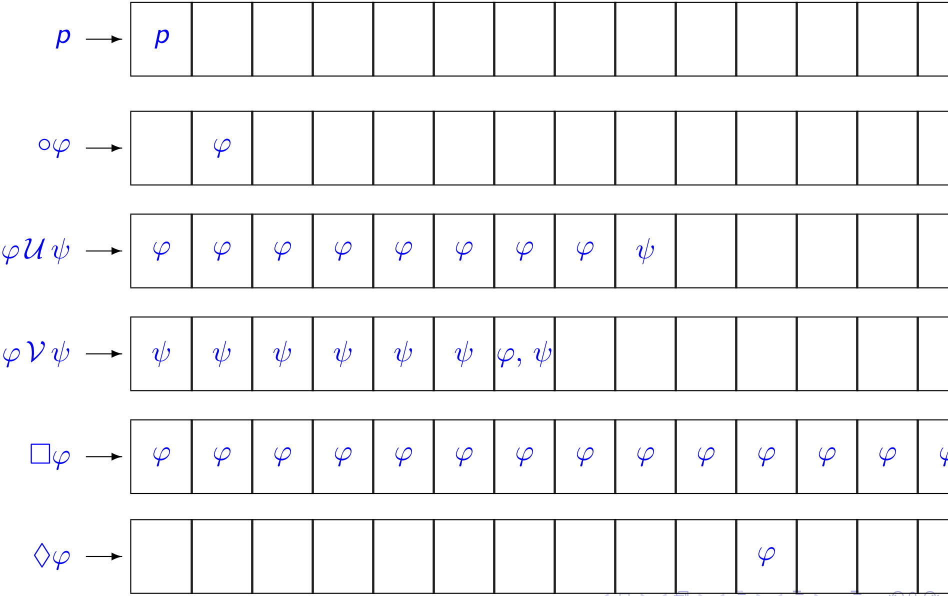
- LTS for traffic light has $3 \times 3 \times 2 = 18$ possible well typed states
  - Is is possible to reach a state where $NSC \neq Red \wedge EWC \neq Red$ from an initial state?
  - If so, what sequence of actions alows this?
  - Do all the immediate predecessors of a state where $NSC = Green \vee EWC = Green$ satisfy $NSC = Red \wedge EWC = Red$?
  - If not, are any of those offend states reachable from and initial state, and if so, how?
- LTS for Mutual Exclusion has $6 \times 6 \times 2 \times 2 = 144$ posible well-tped states.
  - Is is possible to reach a state where $pc1 = m5 \wedge pc2 = n5$?
- How can we state these questions rigorously, formally?
- Can we find an algorihm to answer these types of questions?

Linear Temporal Logic - Syntax

$$\varphi ::= p \mid (\varphi) \mid \neg\varphi \mid \varphi \wedge \varphi' \mid \varphi \vee \varphi'$$
$$\mid \circ\varphi \mid \varphi\,\mathcal{U}\,\varphi' \mid \varphi\,\mathcal{V}\,\varphi' \mid \Box\varphi \mid \Diamond\varphi$$

- p – a propostion over state variables
- $\circ\varphi$ – "next"
- $\varphi\mathcal{U}\varphi'$ – "until"
- $\varphi\mathcal{V}\varphi'$ – "releases"
- $\Box\varphi$ – "box", "always", "forever"
- $\Diamond\varphi$ – "diamond", "eventually", "sometime"

LTL Semantics: The Idea



Formal LTL Semantics

Given:

- $\mathcal{G} = (V, F, af, R, ar)$ signature expressing state propositions
- Q set of states,
- $\mathcal{M}$ modeling function over Q and $\mathcal{G}$: $\mathcal{M}(q, p)$ is true iff q models p. Write $q \models p$.
- $\sigma = q_0 q_1 \ldots q_n \ldots$ infinite sequence of state from Q.
- $\sigma^i = q_i q_{i+1} \ldots q_n \ldots$ the i^{th} tail of σ

Say σ models LTL formula φ, write $\sigma \models \varphi$ as follows:

- $\sigma \models p$ iff $q_0 \models p$
- $\sigma \models \neg\varphi$ iff $\sigma \not\models \varphi$
- $\sigma \models \varphi \wedge \psi$ iff $\sigma \models \varphi$ and $\sigma \models \psi$.
- $\sigma \models \varphi \vee \psi$ iff $\sigma \models \varphi$ or $\sigma \models \psi$.

Formal LTL Semantics

- $\sigma \models \circ\varphi$ iff $\sigma^1 \models \varphi$
- $\sigma \models \varphi\mathcal{U}\psi$ iff for some k, $\sigma^k \models \psi$ and for all $i < k$, $\sigma^i \models \varphi$
- $\sigma \models \varphi\mathcal{V}\psi$ iff for some k, $\sigma^k \models \varphi$ and for all $i \leq k$, $\sigma^i \models \psi$,
  or for all i, $\sigma^i \models \psi$.
- $\sigma \models \Box\varphi$ if for all i, $\sigma^i \models \psi$
- $\sigma \models \Diamond\varphi$ if for some i, $\sigma^i \models \psi$

Some Common Combinations

- $\Box\Diamond p$ "p will hold infinitely often"
- $\Diamond\Box p$ "p will continuously hold from some point on"
- $(\Box p) \Rightarrow (\Box q)$ "if p happens infinitely often, then so does q

Some Equivalences

- $\Box(\varphi \wedge \psi) = (\Box\varphi) \wedge (\Box\psi)$
- $\Diamond(\varphi \vee \psi) = (\Diamond\varphi) \vee (\Diamond\psi)$
- $\Box\varphi = \mathbf{F}\,\mathcal{V}\,\varphi$
- $\Diamond\varphi = \mathbf{T}\,\mathcal{U}\,\varphi$
- $\varphi\,\mathcal{V}\,\psi = \neg((\neg\varphi)\,\mathcal{U}\,(\neg\psi))$
- $\varphi\,\mathcal{U}\,\psi = \neg((\neg\varphi)\,\mathcal{V}\,(\neg\psi))$
- $\neg(\Diamond\varphi) = \Box(\neg\varphi)$
- $\neg(\Box\varphi) = \Diamond(\neg\varphi)$

Some More Eqivalences

- $\Box\varphi = \varphi \wedge \circ\Box\varphi$
- $\Diamond\varphi = \varphi \vee \circ\Diamond\varphi$
- $\varphi\,\mathcal{V}\,\psi = (\varphi \wedge \psi) \vee (\psi \wedge \circ(\varphi\,\mathcal{V}\,\psi))$
- $\varphi\,\mathcal{U}\,\psi = \psi \vee (\varphi \wedge \circ(\varphi\,\mathcal{V}\,\psi)$
- $\Box$, $\Diamond$, $\mathcal{U}$, $\mathcal{V}$ may all be understood recursively, by what they state about right now, and what they state about the future
- Caution: $\Box$ vs $\Diamond$, $\mathcal{U}$ vs $\mathcal{V}$ differ in there limit behavior

Traffic Light Example

Basic Behavior:

- $\Box((NSC = Red) \vee (NSC = Green) \vee (NSC = Yellow))$
- $\Box((NSC = Red) \Rightarrow ((NSC \neq Green) \wedge (NSC \neq Yellow))$
- Similarly for *Green* and *Red*
- $\Box(((NCS = Red) \wedge \circ(NCS \neq Red)) \Rightarrow \circ(NCS = Green))$
- Same as $\Box((NCS = Red) \Rightarrow ((NCS = Red)\,\mathcal{U}\,(NCS = Green)))$
- $\Box(((NCS = Green) \wedge \circ(NCS \neq Green)) \Rightarrow \circ(NCS = Yellow))$
- $\Box(((NCS = Yellow) \wedge \circ(NCS \neq Yellow)) \Rightarrow \circ(NCS = Red))$
- Same for *EWC*

Traffic Light Example

Basic Safety

- $\Box((NSC = Red) \vee (EWC = Red))$
- $\Box(\ ((NSC = Red) \wedge (EWC = Red))\ \mathcal{V}$
  $((NSC \neq Green) \Rightarrow (\circ(NSC = Green))))$

Basic Liveness

- $(\Diamond(NSC = Red)) \wedge (\Diamond(NSC = Green)) \wedge (\Diamond(NSC = Yellow))$
- $(\Diamond(EWC = Red)) \wedge (\Diamond(EWC = Green)) \wedge (\Diamond(EWC = Yellow))$

Proof System for LTL

First step: View $\varphi \mathcal{V} \psi$ as moacro: $\varphi \mathcal{V} \psi = \neg((\neg\varphi) \mathcal{U} (\neg\psi))$

Second Step: Extend all rules of Prop Logic to LTL

Third Step: Add one more rule: $\dfrac{\Box\varphi}{\varphi}$ Gen

Fourth Step: Add a collection of axioms (a sufficient set of 8 exists)

Result: a sound and relatively complete proof system