

Non-standard Model of Propositional Logic

Other models possible

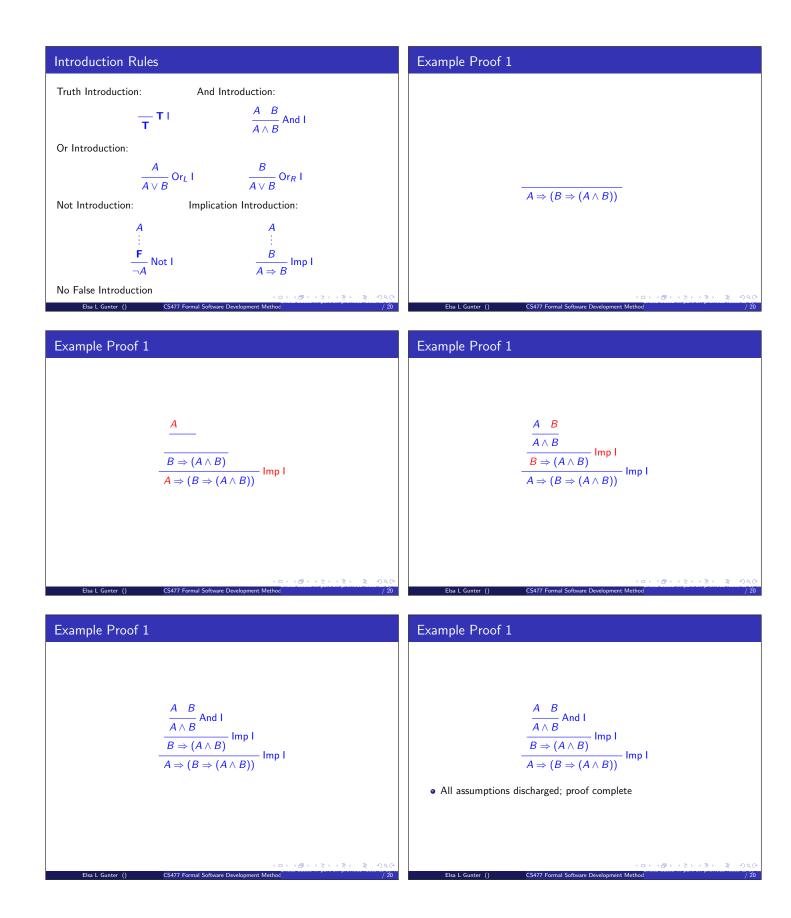
Example:

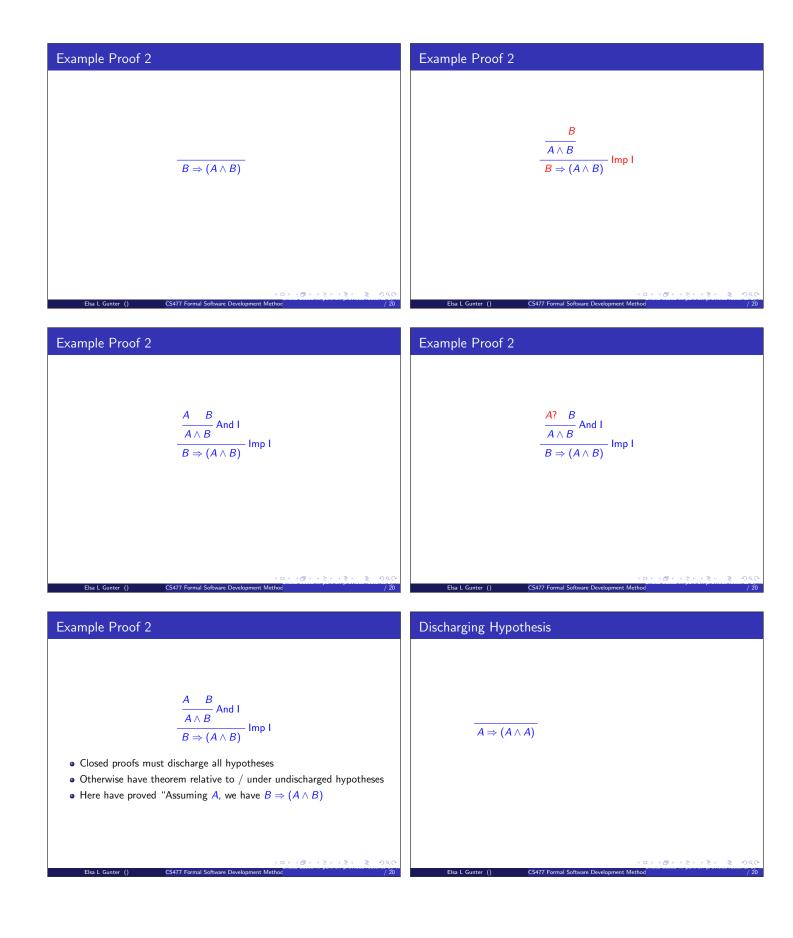
- $\mathcal{C} = \{ true, false, \bot \}$
- Valuations w assign values in C to propositional atoms
- If $\mathcal{J}_w(p) = \bot$ then $\mathcal{J}_w(\neg p) = \bot$, otherwise same as for \mathcal{I}
- $\mathcal{J}_w(p) = \bot$ or $\mathcal{J}_w(q) = \bot$ then $\mathcal{J}_w(\neg p) = \bot$, $\mathcal{J}_w(p \land q) = \bot$, $\mathcal{J}_w(p \lor q) = \bot$, $\mathcal{J}_w(p \Rightarrow q) = \bot$, and $\mathcal{J}_w(p \Leftrightarrow q) = \bot$; otherwise same as for \mathcal{I}
- Note: $A \lor \neg A \not\equiv \mathbf{T}$

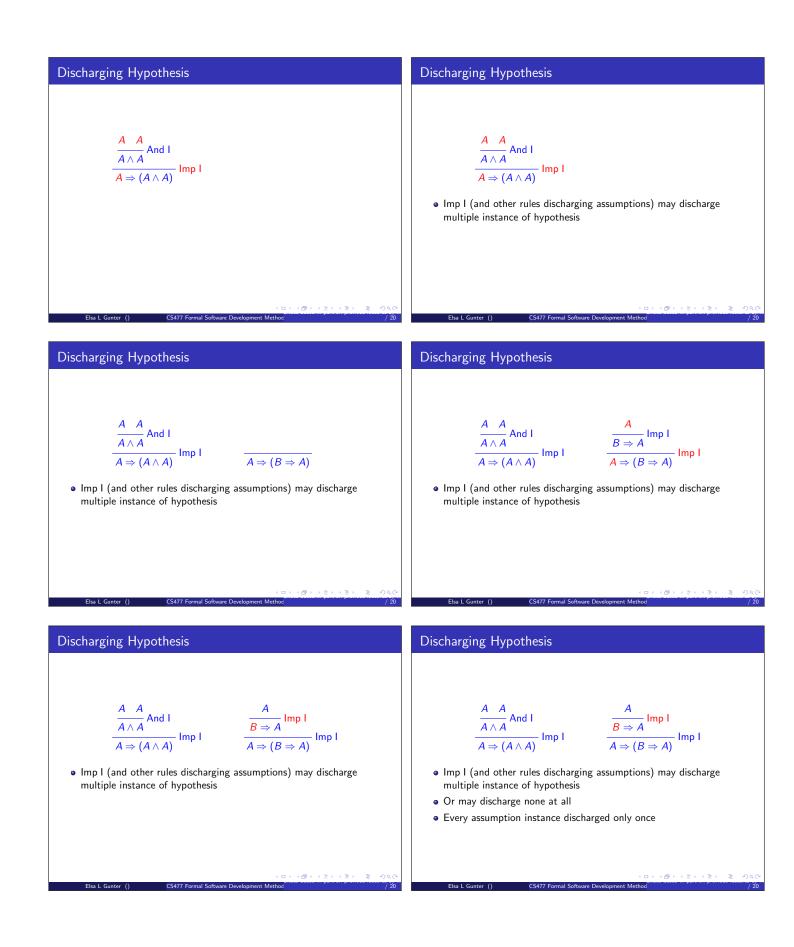
Proofs in Propositional Logic

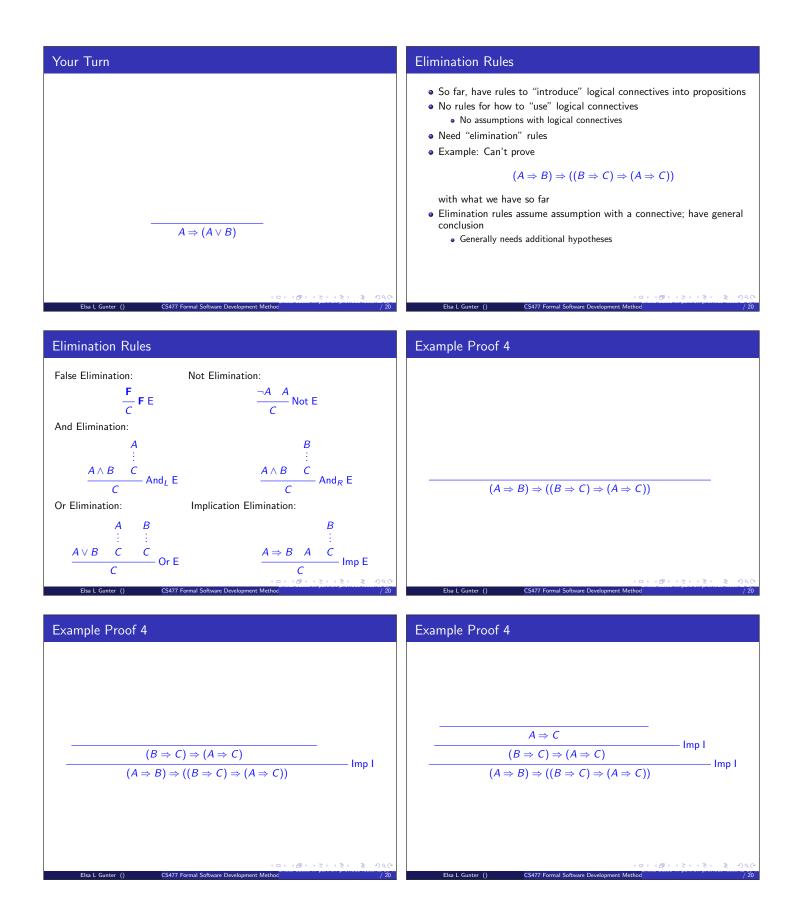
- Natural Deduction proofs are trees with nodes that are inference rules
- Inference rule has hypotheses and conclusion
- Conclusion a single proposition
- Hypotheses zero or more propositions, possibly with hypotheses
- Two main kinds of inference rules:
 - $\bullet\,$ Introduction says how to conclude proposition made from connective is true
 - $\bullet\,$ Eliminations says how to use a proposition made from connective to prove result
- Inference rules associated with connectives
- Rule with no hypotheses called an axiom

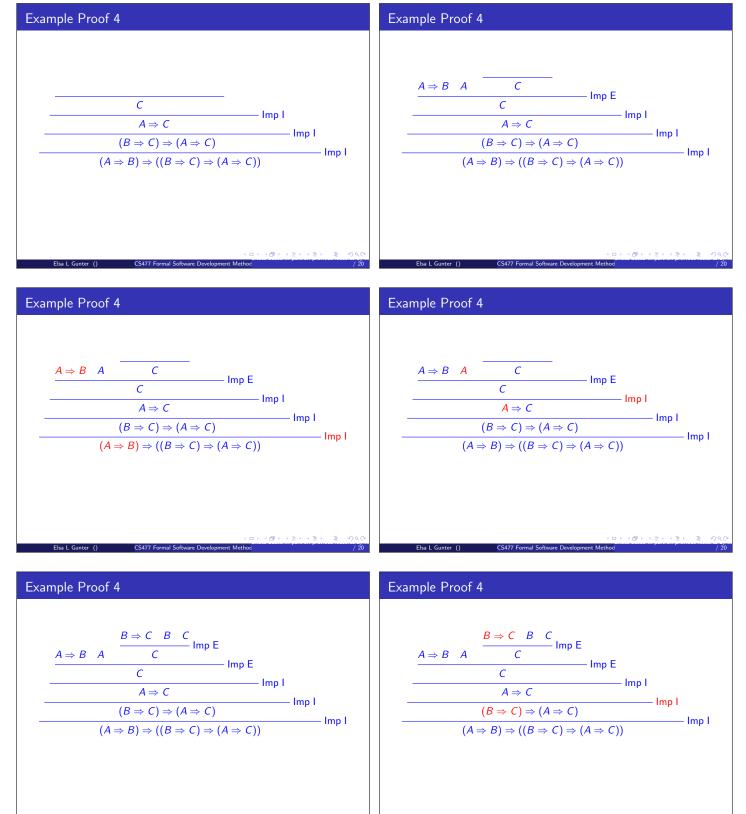
□ ト 4 団 ト 4 Ξ ト 4 Ξ ト Ξ - 9 Q

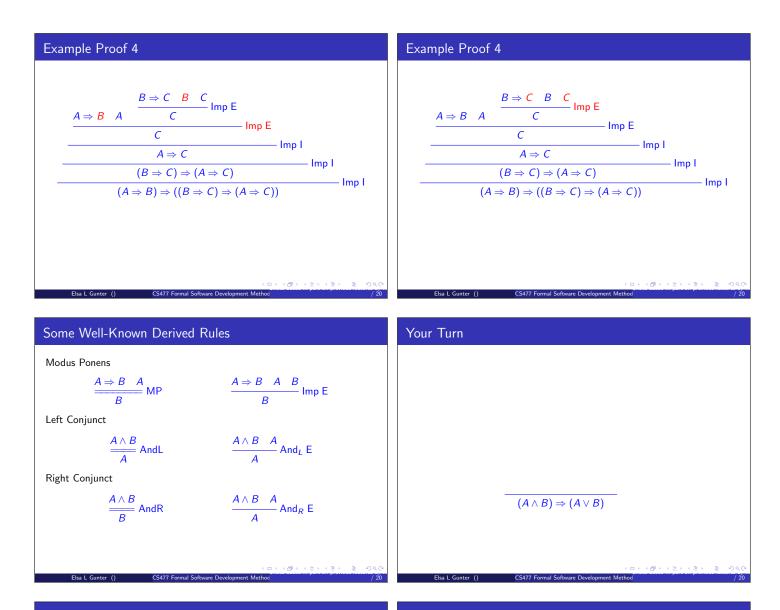










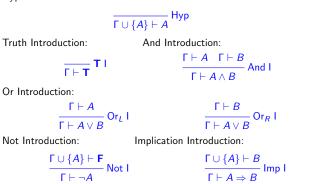


Assumptions in Natural Deduction

- Problem: Keeping track of hypotheses and their discharge in Natural Deduction is *HARD*!
- Solution: Use *sequents* to track hypotheses
- A sequent is a pair of
 - A set of propositions (called assumptions, or hypotheses of sequent) and
 - A proposition (called conclusion of sequent)
- More generally (not here), allow set of hypotheses and set of conclusions



Γ is set of propositions (assumptions/hypotheses) Hypothesis Introduction:



Nat. Ded. Elimination Sequent Rules	Example Proof 4, Revisited
F is set of propositions (assumptions/hypotheses) Not Elimination: Implication Elimination:	
$\frac{\Gamma \vdash \neg A \Gamma \vdash A}{\Gamma \vdash C} \operatorname{Not} E \qquad \frac{\Gamma \vdash A \Rightarrow B \Gamma \vdash A \Gamma \cup \{B\} \vdash C}{\Gamma \vdash C} \operatorname{Imp} E$	
And Elimination:	
$\frac{\Gamma \vdash A \land B \Gamma \cup \{A\} \vdash C}{\Gamma \vdash C} \operatorname{And}_{L} E \qquad \frac{\Gamma \vdash A \land B \Gamma \cup \{B\} \vdash C}{\Gamma \vdash C} \operatorname{And}_{R} E$	
False Elimination: Or Elimination:	
$\frac{\Gamma \vdash \mathbf{F}}{\Gamma \vdash C} \mathbf{F} \mathbf{E} \qquad \frac{\Gamma \vdash A \lor B \Gamma \cup \{A\} \vdash C \Gamma \cup \{B\} \vdash C}{\Gamma \vdash C} \text{ Or } \mathbf{E}$	
シング・ボー・バー・パー・	(日) (月) (日) (日) (日) (日) (日) (日) (日) (日) (日) (日
Elsa L Gunter () CS477 Formal Software Development Method / 20	Elsa L Gunter () CS477 Formal Software Development Method / 20