Propostional Logic Inference Rules, Sequent Style

Introduction Rules

Γ is set of propositions (assumptions/hypotheses)

Hypothesis Introduction:

$$\frac{}{\Gamma \cup \{A\} \vdash A}\ \textsf{Hyp}$$

Truth Introduction:

$$\frac{}{\Gamma \vdash \mathbf{T}}\ \textsf{T I}$$

And Introduction:

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B}\ \textsf{And I}$$

Or Introduction:

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}\ \textsf{Or}_L\ \textsf{I} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B}\ \textsf{Or}_R\ \textsf{I}$$

Not Introduction:

$$\frac{\Gamma \cup \{A\} \vdash \mathbf{F}}{\Gamma \vdash \neg A}\ \textsf{Not I}$$

Implication Introduction:

$$\frac{\Gamma \cup \{A\} \vdash B}{\Gamma \vdash A \Rightarrow B}\ \textsf{Imp I}$$

Elimination Rules

False Elimination:

$$\frac{\mathbf{F}}{C}\ \textsf{F E}$$

Not Elimination:

$$\frac{\neg A \quad A}{C}\ \textsf{Not E}$$

And Elimination:

$$\frac{A \wedge B \quad \begin{matrix} A \\ \vdots \\ C \end{matrix}}{C}\ \textsf{And}_L\ \textsf{E} \qquad \frac{A \wedge B \quad \begin{matrix} B \\ \vdots \\ C \end{matrix}}{C}\ \textsf{And}_R\ \textsf{E}$$

Or Elimination:

$$\frac{A \vee B \quad \begin{matrix} A \\ \vdots \\ C \end{matrix} \quad \begin{matrix} B \\ \vdots \\ C \end{matrix}}{C}\ \textsf{Or E}$$

Implication Elimination:

$$\frac{A \Rightarrow B \quad A \quad \begin{matrix} B \\ \vdots \\ C \end{matrix}}{C}\ \textsf{Imp E}$$

First Order Logic

All rules from Propositional Logic included

$$\frac{\Gamma \vdash \psi[t/x]}{\Gamma \vdash \exists x.\psi}\ \textsf{Ex I} \qquad \frac{\Gamma \vdash \exists x.\psi \quad \Gamma \cup \{(\psi[y/x])\} \vdash \varphi}{\Gamma \vdash \varphi}\ \textsf{Ex E}$$

provided
$y \notin fv(\varphi) \cup (fv(\psi) \setminus \{x\}) \cup \bigcup_{\psi' \in \Gamma} fv(\psi')$

$$\frac{\Gamma \vdash \psi[y/x]}{\Gamma \vdash \forall x.\psi}\ \textsf{All I} \qquad \frac{\Gamma \vdash \forall x.\psi \quad \Gamma \cup \{\psi[t/x]\} \vdash \varphi}{\Gamma \vdash \varphi}\ \textsf{All E}$$

provided
$y \notin (fv(\psi) \setminus \{x\}) \cup \bigcup_{\psi' \in \Gamma} fv(\psi')$

Floyd-Hoare Logic

Assingment Rule

$$\frac{}{\{\!|P[e/x]|\!\}\ x\ :=\ e\ \{\!|P|\!\}}$$

Rule of Consequence

$$\frac{P \Rightarrow P' \quad \{\!|P'|\!\}\ C\ \{\!|Q'|\!\} \quad Q' \Rightarrow Q}{\{\!|P|\!\}\ C\ \{\!|Q|\!\}}$$

Sequencing Rule

$$\frac{\{\!|P|\!\}\ C_1\ \{\!|Q|\!\} \quad \{\!|Q|\!\}\ C_2\ \{\!|R|\!\}}{\{\!|P|\!\}\ C_1;\ C_2\ \{\!|R|\!\}}$$

If Then Else Rule

$$\frac{\{\!|P \wedge B|\!\}\ C_1\ \{\!|Q|\!\} \quad \{\!|P \wedge \neg B|\!\}\ C_2\ \{\!|Q|\!\}}{\{\!|P|\!\}\ if\ B\ then\ C_1\ else\ C_2\ \{\!|Q|\!\}}$$

While Rule

$$\frac{\{\!|P \wedge B|\!\}\ C\ \{\!|P|\!\}}{\{\!|P|\!\}\ while\ B\ do\ C\ \{\!|P \wedge \neg B|\!\}}$$

Weakest Precondition and Verification Condition Generation

$\textsf{wp}\ (x := e)\ Q = Q[x \Rightarrow e]$
$\textsf{wp}\ (C_1; C_2)\ Q = \textsf{wp}\ C_1\ (\textsf{wp}\ C_2\ Q)$
$\textsf{wp}\ (\texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2\ \texttt{fi})\ Q =$
$\quad (B \wedge (\textsf{wp}\ C_1\ Q)) \vee ((\neg B) \wedge (\textsf{wp}\ C_2\ Q))$
$\textsf{wp}\ (\texttt{while}\ B\ \texttt{inv}\ P\ \texttt{do}\ C\ \texttt{od})\ Q = P$

$\textsf{vcg}\ (x := e)\ Q = \text{true}$
$\textsf{vcg}\ (C_1; C_2)\ Q = (\textsf{vcg}\ C_1\ (\textsf{wp}\ C_2\ Q)) \wedge (\textsf{vcg}\ C_2\ Q)$
$\textsf{vcg}\ (\texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2\ \texttt{fi})\ Q = (\textsf{vcg}\ C_1\ Q) \wedge (\textsf{vcg}\ C_2\ Q)$
$\textsf{vcg}\ (\texttt{while}\ B\ \texttt{inv}\ P\ \texttt{do}\ C\ \texttt{od})\ Q =$
$\quad ((P \wedge B) \Rightarrow (\textsf{wp}\ C\ P)) \wedge (\textsf{vcg}\ C\ P) \wedge ((P \wedge (\neg B)) \Rightarrow Q)$