CS 473: Algorithms, Fall 2010
HBS 9: Midterm Review

Problem 1. **[The Problem with Change]**
Consider the problem of making change for n cents using the least number of coins.

1. Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution.
2. Suppose that the available coins have the values $c^0, c^1, ..., c^k$ for some integers $c > 1$ and $k \geq 1$. Show that the greedy algorithm always yields an optimal solution.
3. Give a set of 4 coin values for which the greedy algorithm does not yield an optimal solution, show why.
4. Give a dynamic programming algorithm that yields an optimal solution for an arbitrary set of coin values.

Problem 2. **[Small Changes to MST]**
Let G be a connected, undirected graph where each edge e has weight $w(e)$. You may assume all edge weights are positive and distinct. Consider a Minimum Spanning Tree T of G. Suppose that we decrease one of the edges not in T to a new distinct, positive value. How could you find the MST in the modified graph?

Problem 3. **[Flow Facts?]**
Which of the following statements are true and which are false? Justify your answer.

1. If all directed edges in a network have distinct capacities, then there is a unique max flow.
2. Consider a graph $G = (V, E)$. Now, for each edge $e = (u, v)$ with capacity $c(e)$, we will add an edge $e' = (v, u)$ in the opposite direction with the same capacity $c(e)$. This alteration to G will not change the value of the max flow.

Problem 4. **[Randomized Max Cut]**
Consider the *Max Cut* problem: given an undirected graph $G = (V, E)$ and weight function $w : E \to Z^+$, find a cut (A, B) such that the value of the weights across the cut is *maximized*. We will now analyze a simple randomized algorithm for this problem:

For each v, independently put it in A with probability 1/2. Output the cut $(A, V \setminus A)$.

1. What is the probability of edge (u, v) being in the cut?
2. What is the expected weight of the edges in the cut?
3. What is the maximum weight of any cut?