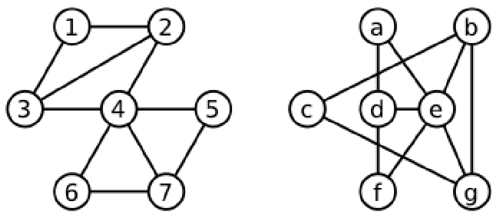
CS 473: Algorithms, Fall 2010
HBS 12

Problem 1. [**Graph Isomorphism**]
Two graphs are said to be **isomorphic** if one can be transformed into the other just by relabeling the vertices. For example, the graphs shown below are isomorphic; the left graph can be transformed into the right graph by the relabeling $(1, 2, 3, 4, 5, 6, 7) \implies (c, g, b, e, a, f, d)$.



Two isomorphic graphs.

Consider the following related decision problems:

- GraphIsomorphism: Given two graphs G and H, determine whether G and H are isomorphic.
- EvenGraphIsomorphism: Given two graphs G and H, such that every vertex in G and H has even degree, determine whether G and H are isomorphic.
- SubgraphIsomorphism: Given two graphs G and H, determine whether G is isomorphic to a subgraph of H.

a.) Describe a polynomial-time reduction from GraphIsomorphism to EvenGraphIsomorphism.

b.) Describe a polynomial-time reduction from GraphIsomorphism to SubgraphIsomorphism.

c.) Prove that SubgraphIsomorphism is NP-complete by reducing from Clique.

Problem 2. [**Self-Reductions**]
In each case below, assume that you are given a black box which can answer the decision version of the indicated problem. Use a polynomial number of calls to the black box to construct the desired set.

- Independent set: Given a graph G and an integer k, does G have a subset of k vertices that are pairwise nonadjacent?
- Subset sum: Given a multiset (elements can appear more than once) $X = x_1, \ldots, x_k$ of positive integers, and a positive integer S, does there exist a subset of X with sum exactly S?
- k-Color: Given a graph G, is there a proper k-coloring? In other words, can we assign one of the k colors to each node such that no node is adjacent to a node of the same color?