# CS 473: Algorithms, Fall 2009
# HW 10 (due Thursday, December 3rd in class)

This homework contains three problems. **Read the instruction for submitting homework on the course web page**. In particular, *make sure* that you write the solutions for the problems on separate sheets of paper. Write your name and netid on each sheet.

**Collaboration Policy:** For this home work, students can work in groups of up to 3 members each. Each group submits only one written solution (some groups will do an oral presentation. Indicate your group members on the homework (netids are needed)).

1. (35 pts) Describe a polynomial time reduction from the 3-Color problem (given a graph $G$, is it 3-colorable) to 3-SAT.

2. (30 pts) A kite is a graph on an even number of nodes, say $2n$, in which $n$ of the nodes form a clique and the remaining $n$ vertices are connected in a "tail" that consists of a path joined to one of the nodes in the clique. Given a graph $G$ and an integer $k$, the KITE problem asks for a subgraph which is a kite that contains $2k$ nodes. Prove that KITE is NP-Complete.

3. (35 pts) Given an undirected graph $G = (V, E)$ and subset of nodes $S \subseteq V$ called terminals, a Steiner tree for $S$ in $G$ is a tree $T = (V', E')$ such that $T$ is a subgraph of $G$ and $T'$ contains all the terminals. A node $v \in V' \setminus S$ is said to be a Steiner node. Consider the following decision problem: given a graph $G = (V, E)$, a set of terminals $S \subseteq V$, and an integer $k$, is there a Steiner tree for $S$ in $G$ that contains at most $k$ Steiner nodes? Prove that this problem is NP-complete. *Hint:* Use a reduction from the Set Cover problem; create a bipartite graph between sets and elements.

4. (0 pts) The Halting problem is the following. Given a Turing Machine (or a program) $M$ and a string $w$, does $M$ halt on $w$? Prove that the Halting problem is NP-hard.