Lecture 3: Direct Link Networks

CS/ECE 438: Communication Networks

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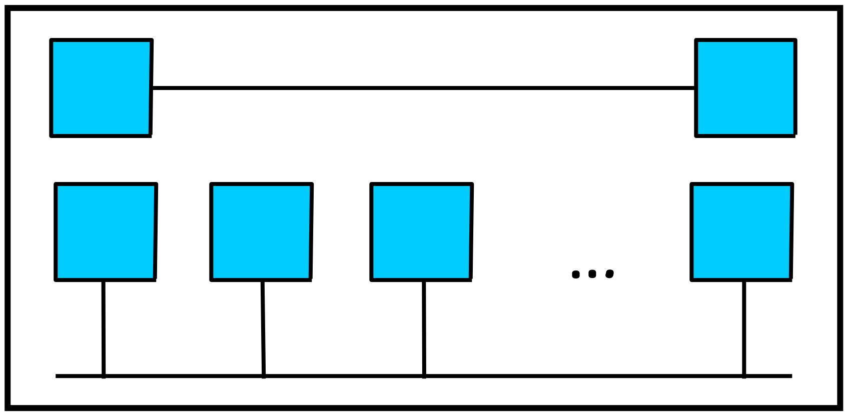
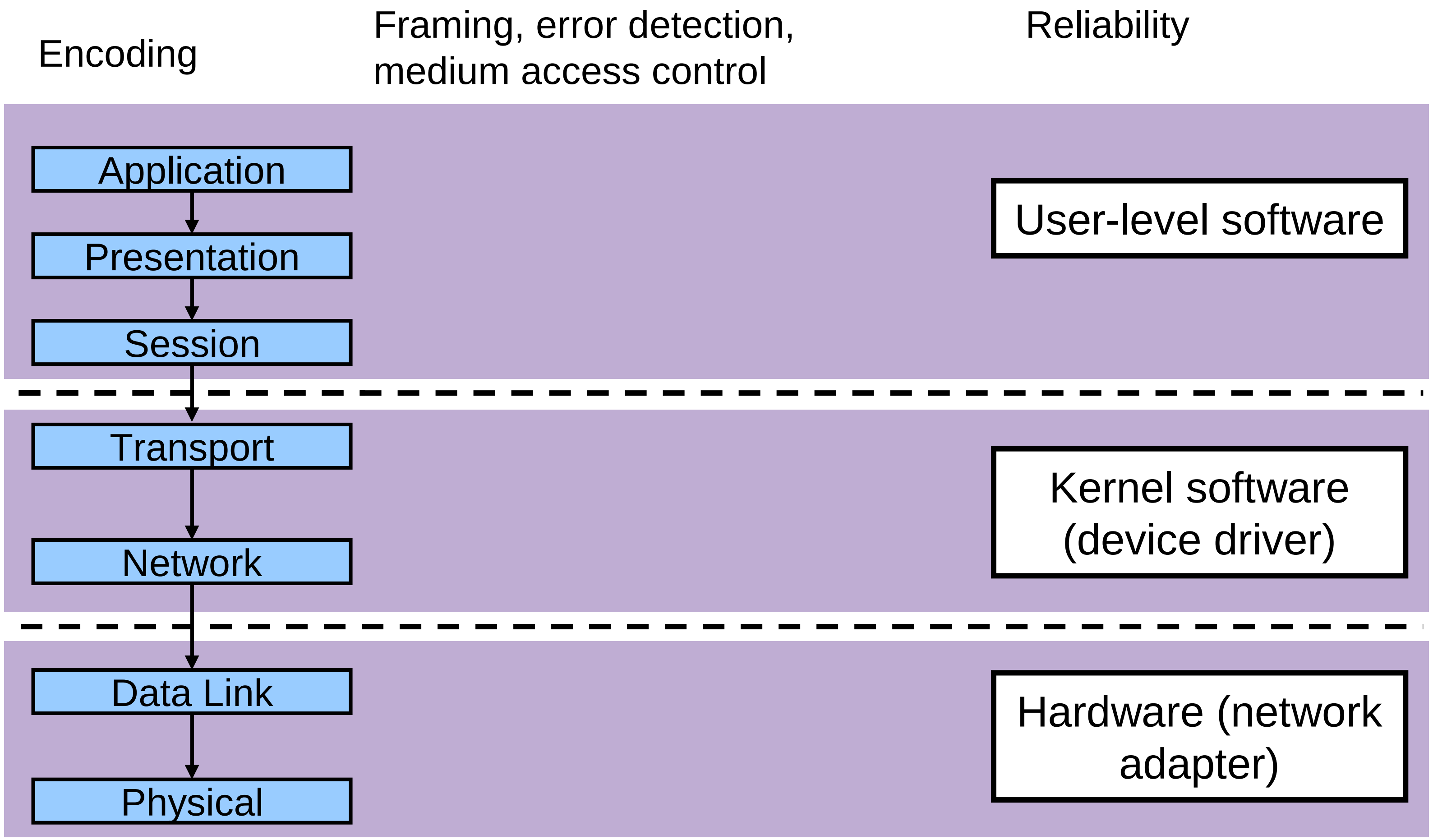
Direct Link Networks

- All hosts are directly connected by a physical medium
- Key points
  - Encoding and Modulation
  - Framing
  - Error Detection
  - Reliable Transmission
  - Medium Access Control



Internet Protocols



Direct Link Networks - Outline

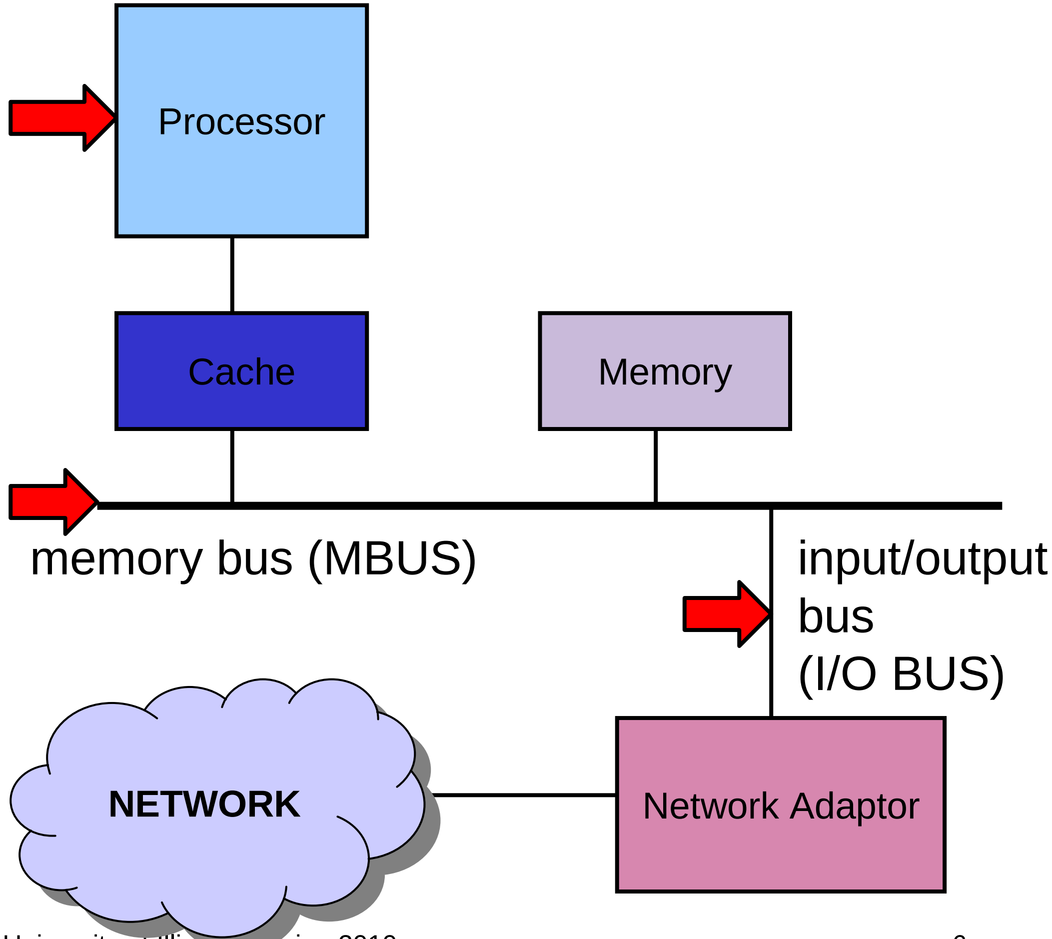
- Hardware building blocks
- Encoding
- Framing
- Error detection
- Reliable transmission
- Multiple access media (MAC examples)
- Network adapters

Hardware Building Blocks

- Nodes
  - Hosts: general purpose computers
  - Switches: typically special purpose hardware
  - Routers: varied

Nodes: Workstation Architecture

- Finite memory
  - Scarce resource
- Generally limited by bus speeds, NOT processor speeds



Hardware Building Blocks

- Links
  - Physical medium carrying
  - Media
    - Copper wire with electronic signaling
    - Glass fiber with optical signaling
    - Wireless with electromagnetic (radio, infrared, microwave) signaling

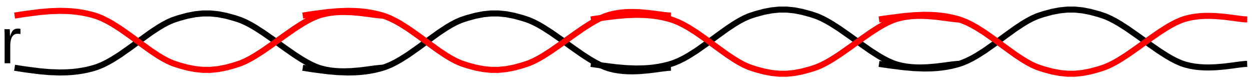
Links - Copper

- Copper-based Media

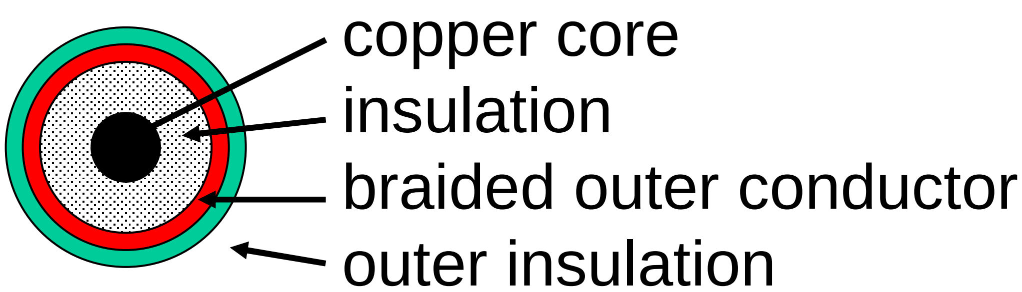
Medium	Bandwidth	Distance
Category 3 Twisted Pair		
Category 5 Twisted Pair	10-100Mbps	100m
ThinNet Coaxial Cable	10-100Mbps	200m
ThickNet Coaxial Cable	10-100Mbps	500m

more twists, less crosstalk, better signal over longer distances

twisted pair



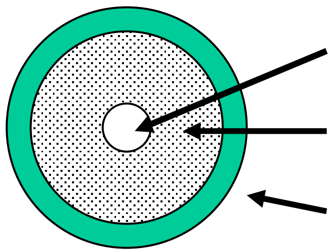
coaxial cable (coax)



More expensive than twisted pair
High bandwidth and excellent noise immunity

Links - Optical

- Optical Media
  - Multimode Fiber 100Mbps 2km
  - Single Mode Fiber 100-2400Mbps 40km



Links - Optical

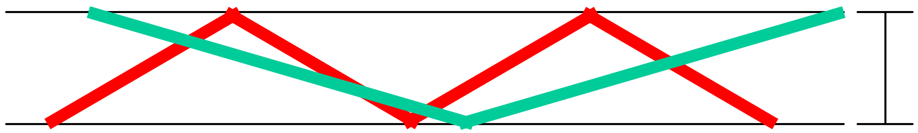
- Single mode fiber
  - Expensive to drive (Lasers)
  - Lower attenuation (longer distances) ≤ 0.5 dB/km
  - Lower dispersion (higher data rates)

- Multimode fiber
  - Cheap to drive (LED's)
  - Higher attenuation
  - Easier to terminate

core of single mode fiber

~1 wavelength thick = ~1 micron

core of multimode fiber (same frequency; colors for clarity)



O(100 microns) thick

Links - Optical

- Advantages of optical communication
  - Higher bandwidths
  - Superior attenuation properties
  - Immune from electromagnetic interference
  - No crosstalk between fibers
  - Thin, lightweight, and cheap (the fiber, not the optical-electrical interfaces)

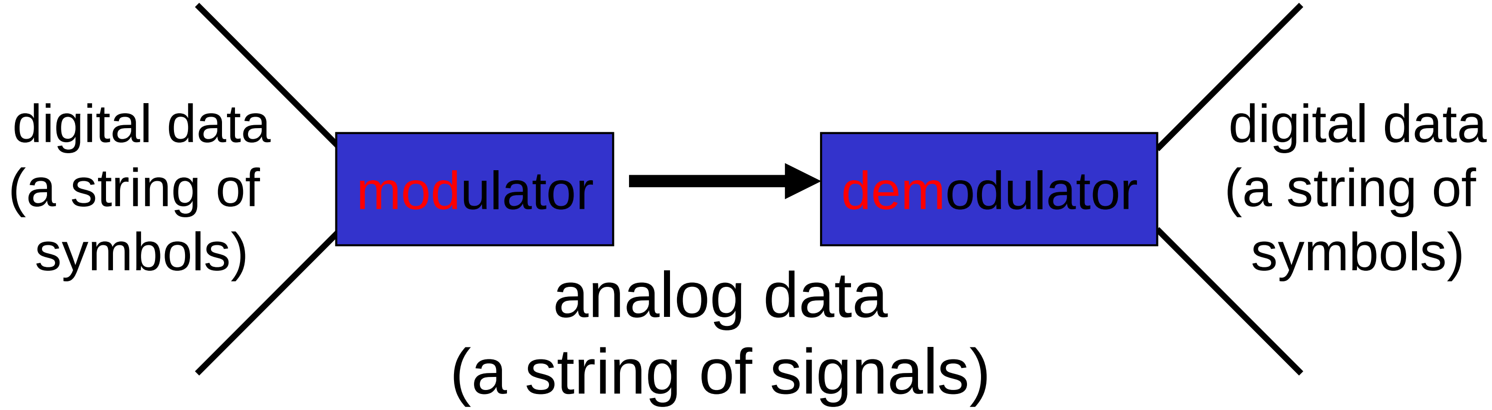
Leased Lines

• POTS	64Kbps
• ISDN	128Kbps
• ADSL	1.5-8Mbps/16-640Kbps
• Cable Modem	0.5-2Mbps
• DS1/T1	1.544Mbps
• DS3/T3	44.736Mbps
• STS-1	51.840Mbps
• STS-12 (ATM rate)	622.080Mbps (ATM)
• OC-48	2.5 Gbps
• OC-192	10 Gbps

Wireless

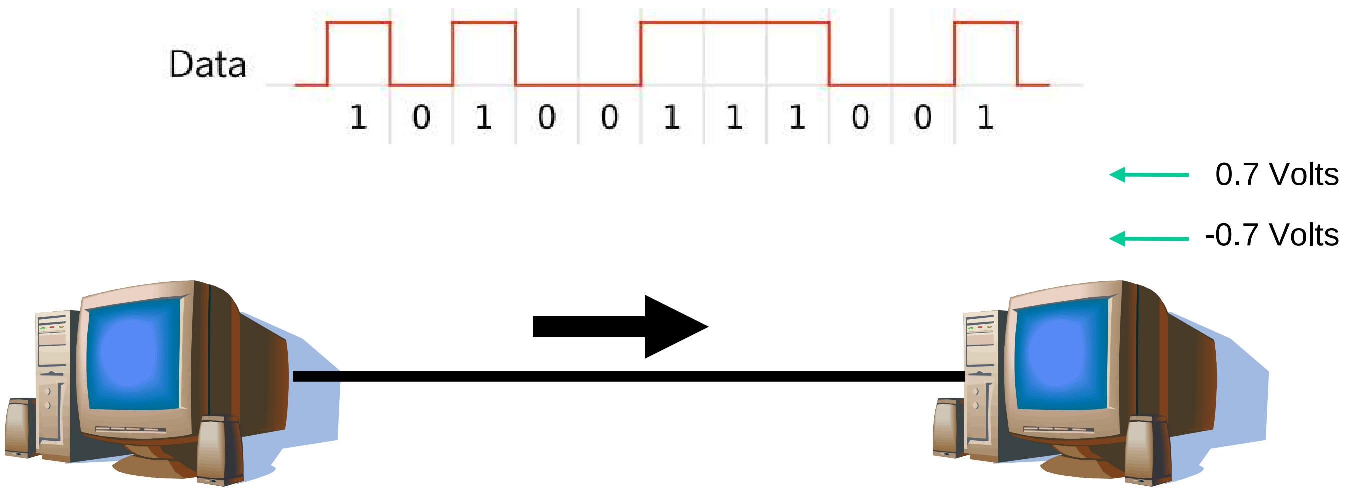
- Cellular
  - AMPS 13Kbps 3km
  - PCS, GSM 300Kbps 3km
- Wireless Local Area Networks (WLAN)
  - Infrared 4Mbps 10m
  - 900Mhz 2Mbps 150m
  - 2.4GHz 2Mbps 150m
  - 2.4Ghz 11Mbps 80m
  - 2.4Ghz 54Mbps 75m
  - 5Ghz 54Mbps 30m
  - Bluetooth 700Kbps 10m
- Satellites
  - Geosynchronous satellite 600-1000 Mbps continent
  - Low Earth orbit (LEO) ~400 Mbps world

Encoding



- Problems with signal transmission
  - Attenuation: Signal power absorbed by medium
  - Dispersion: A discrete signal spreads in space
  - Noise: Random background "signals"

How can two hosts communicate?



- Encode information on modulated "Carrier signal"
  - Phase, frequency, and amplitude modulation, and combinations thereof
  - Ethernet: self-clocking Manchester coding ensures one transition per clock
  - Technologies: copper, optical, wireless

Encoding

- Goal
  - Understand how to connect nodes in such a way that bits can be transmitted from one node to another
- Idea
  - The physical medium is used to propagate signals
    - Modulate electromagnetic waves
    - Vary voltage, frequency, wavelength
  - Data is encoded in the signal

Analog vs. Digital Transmission

- Analog and digital correspond roughly to continuous and discrete
- Data: entities that convey meaning
  - Analog: continuously varying patterns of intensity (e.g., voice and video)
  - Digital: discrete values (e.g., integers, ASCII text)
- Signals: electric or electromagnetic encoding of data
  - Analog: continuously varying electromagnetic wave
    - May be propagated over a variety of medium
  - Digital: sequence of voltage pulses
    - May be transmitted over a wire medium

Analog vs. Digital Transmission

- Advantages of digital transmission over analog
  - Cheaper
  - Suffers more attenuation
    - But reasonably low-error rates over arbitrary distances
    - Calculate/measure effects of transmission problems
    - Periodically interpret and regenerate signal
  - Simpler for multiplexing distinct data types (audio, video, e-mail, etc.)
  - Easier to encrypt
- Two examples based on modulator-demodulators (modems)
  - Electronic Industries Association (EIA) standard: RS-232
  - International Telecommunications Union (ITU)
    V.32 9600 bps modem standard

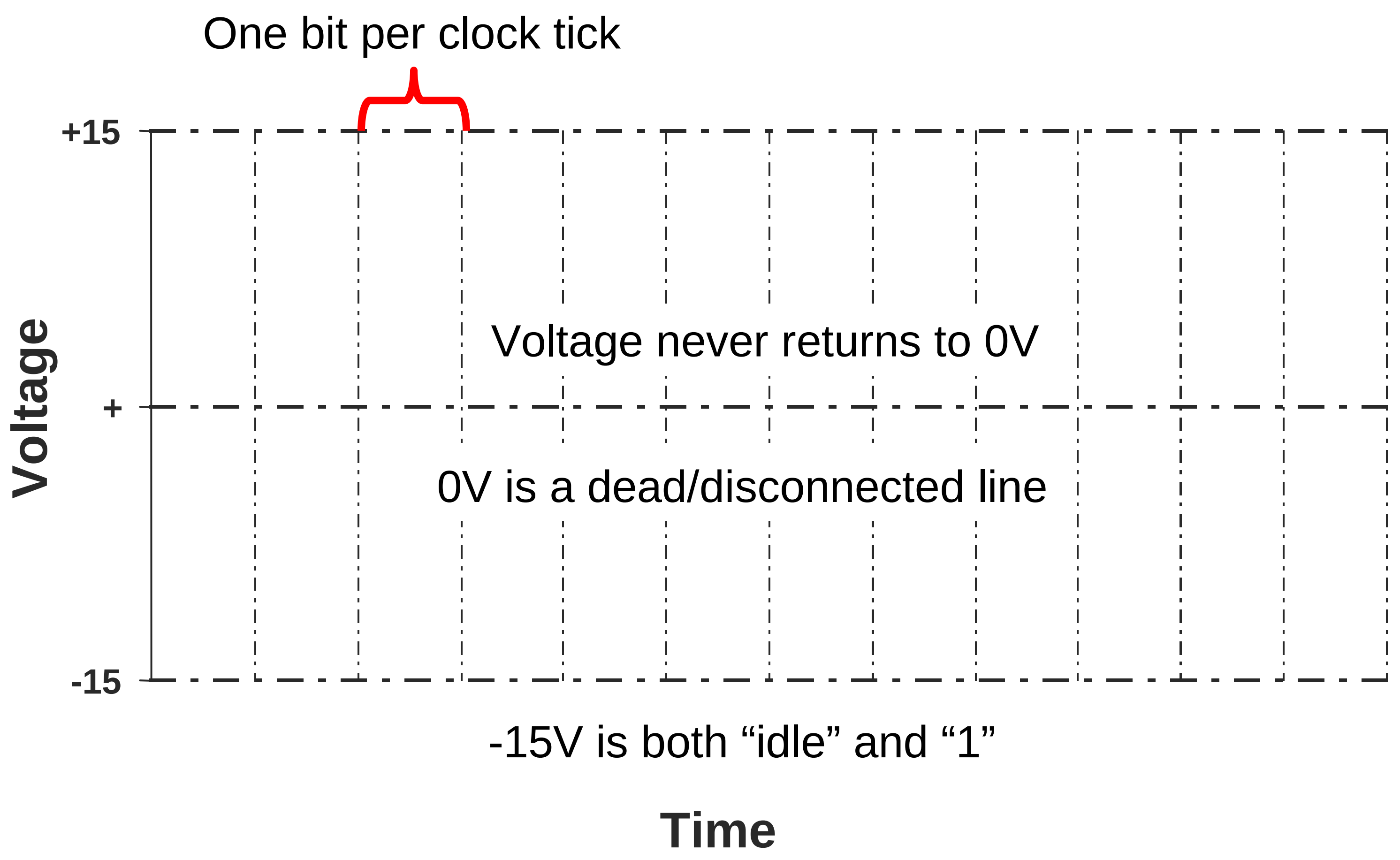
Bauds and Bits

- Baud rate
  - Number of symbols transmitted per second
- Bit rate
  - Actual number of bits transmitted per second
- Relationship
  - Depends on the number of bits encoded in each symbol

RS-232

- Communication between computer and modem
- Uses two voltage levels (+15V, -15V), a binary voltage encoding
- Data rate limited to 19.2 kbps (RS-232-C); raised in later standards
- Characteristics
  - Serial
    - One signaling wire, one bit at a time
  - Asynchronous
    - Line can be idle, clock generated from data
  - Character-based
    - Send data in 7- or 8-bit characters

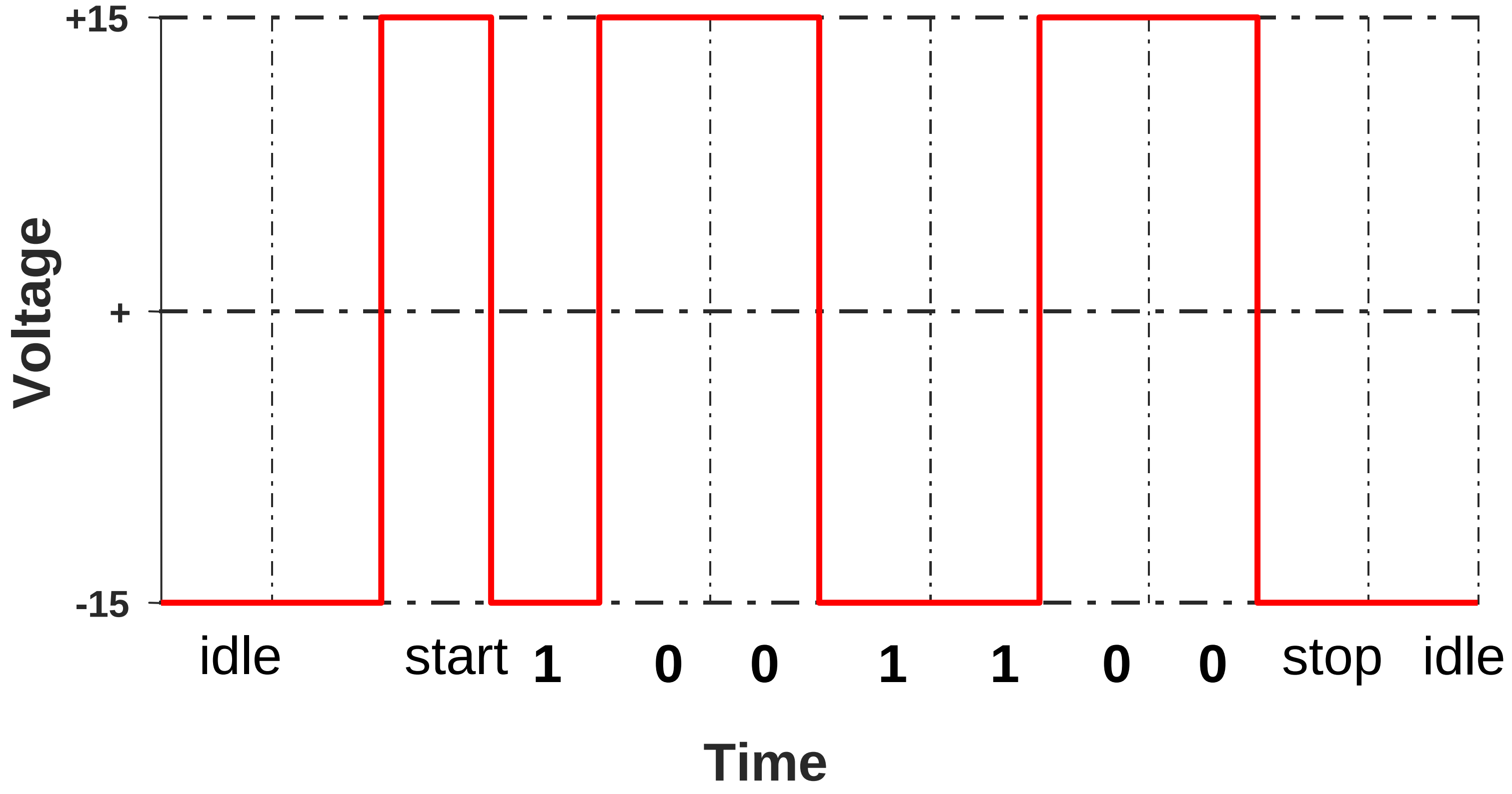
RS-232 Timing Diagram



RS-232

- Initiate send by
  - Push to 15V for one clock (start bit)
- Minimum delay between character transmissions
  - Idle for one clock at -15V (stop bit)
- One character
  - 2+ voltage transitions
- Total Bits
  - 9 bits for 7 bits of data (78% efficient)
- Start and stop bits also provide framing

RS-232 Timing Diagram

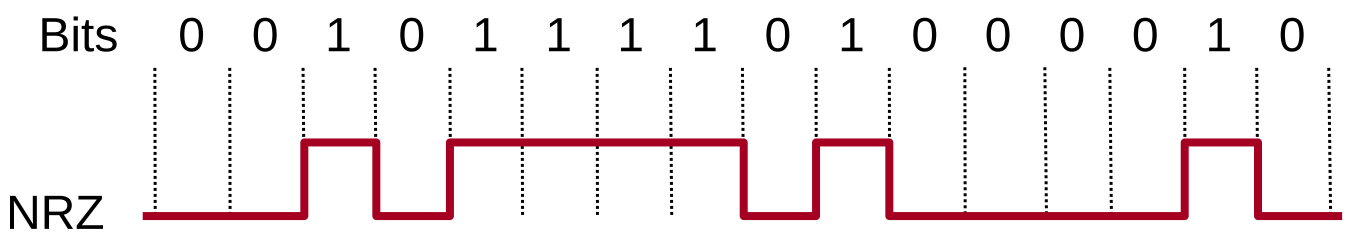


Voltage Encoding

- Binary voltage encoding
  - Done with RS-232 example
  - Generalize before continuing with V.32 (not a binary voltage encoding)
- Common binary voltage encodings
  - Non-return to zero (NRZ)
  - NRZ inverted (NRZI)
  - Manchester (used by IEEE 802.3—10 Mbps Ethernet)
  - 4B/5B

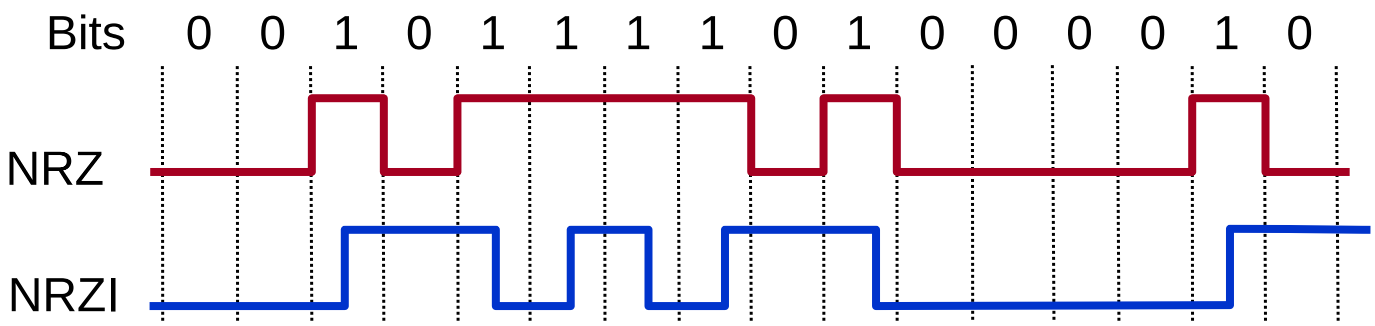
Non-Return to Zero (NRZ)

- Signal to Data
  - High ⇨ 1
  - Low ⇨ 0
- Comments
  - Transitions maintain clock synchronization
  - Long strings of 0s confused with no signal
  - Long strings of 1s causes baseline wander
  - Both inhibit clock recovery



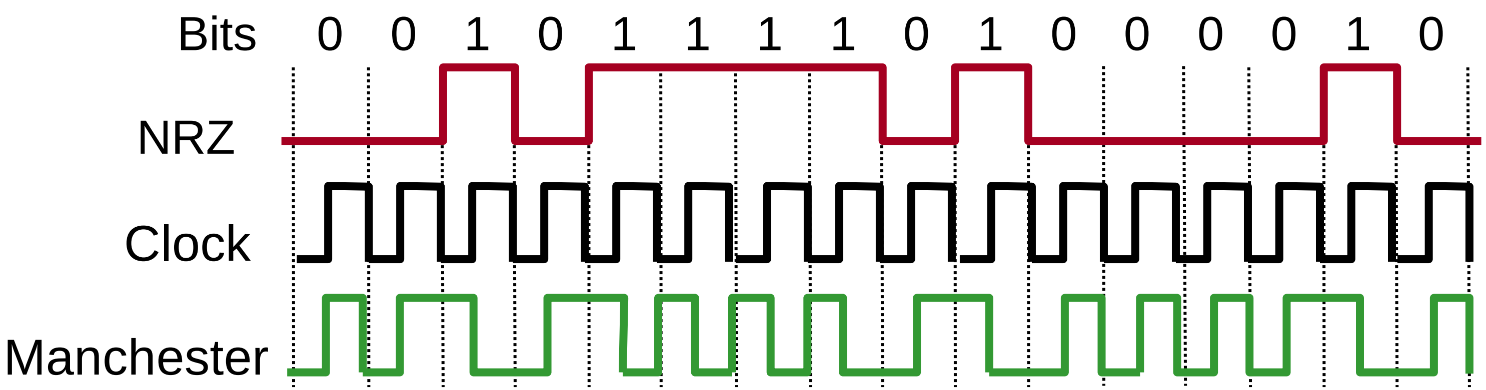
Non-Return to Zero Inverted (NRZI)

- Signal to Data
  - Transition ⇨ 1
  - Maintain ⇨ 0
- Comments
  - Solves series of 1s, but not 0s



Manchester Encoding

- Signal to Data
  - XOR NRZ data with clock
  - High to low transition ⇨ 1
  - Low to high transition ⇨ 0
- Comments
  - (used by IEEE 802.3—10 Mbps Ethernet)
  - Solves clock recovery problem
  - Only 50% efficient (½ bit per transition)



4B/5B

- Signal to Data
  - Encode every 4 consecutive bits as a 5 bit symbol
- Symbols
  - At most 1 leading 0
  - At most 2 trailing 0s
  - Never more than 3 consecutive 0s
  - Transmit with NRZI
- Comments
  - 16 of 32 possible codes used for data
  - At least two transitions for each code
  - 80% efficient

4B/5B – Data Symbols

At most 1 leading 0	At most 2 trailing 0s
0000 ⇒ 11110	1000 ⇒ 10010
0001 ⇒ 01001	1001 ⇒ 10011
0010 ⇒ 10100	1010 ⇒ 10110
0011 ⇒ 10101	1011 ⇒ 10111
0100 ⇒ 01010	1100 ⇒ 11010
0101 ⇒ 01011	1101 ⇒ 11011
0110 ⇒ 01110	1110 ⇒ 11100
0111 ⇒ 01111	1111 ⇒ 11101

4B/5B – Control Symbols

- 11111 ⇒ idle
- 11000 ⇒ start of stream 1
- 10001 ⇒ start of stream 2
- 01101 ⇒ end of stream 1
- 00111 ⇒ end of stream 2
- 00100 ⇒ transmit error
- Other ⇒ invalid

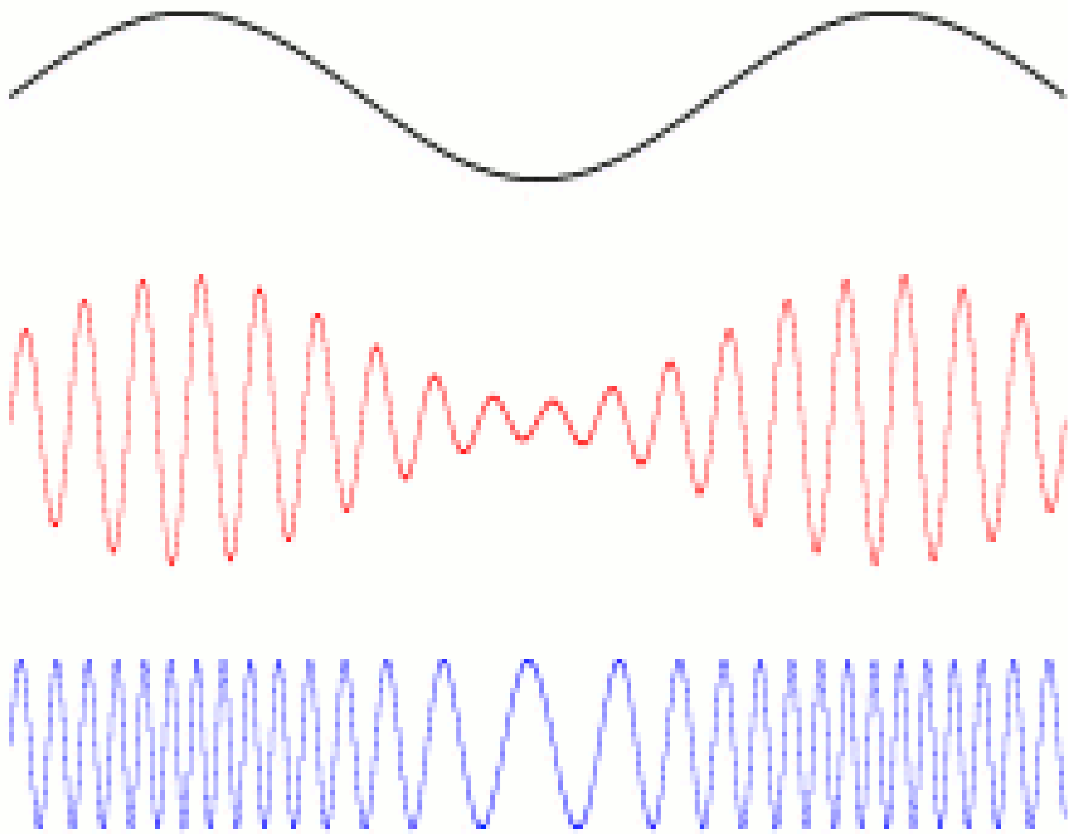
- *Handout: Problem 1*

Binary Voltage Encodings

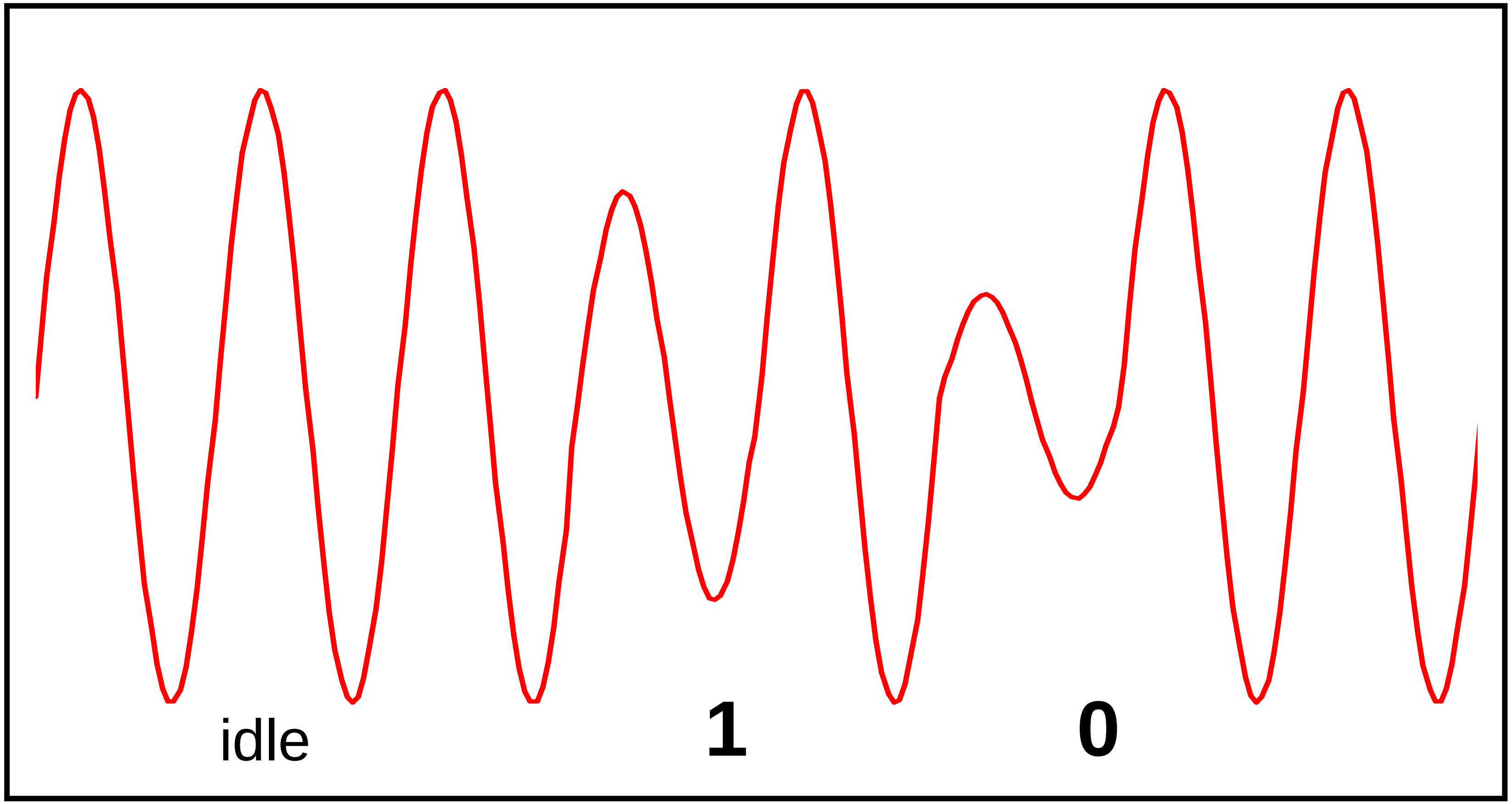
- Problem with binary voltage (square wave) encodings
  - Wide frequency range required, implying
    - Significant dispersion
    - Uneven attenuation
  - Prefer to use narrow frequency band (carrier frequency)
- Types of modulation
  - Amplitude (AM)
  - Frequency (FM)
  - Phase/phase shift
  - Combinations of these

Example:
AM/FM for continuous signal

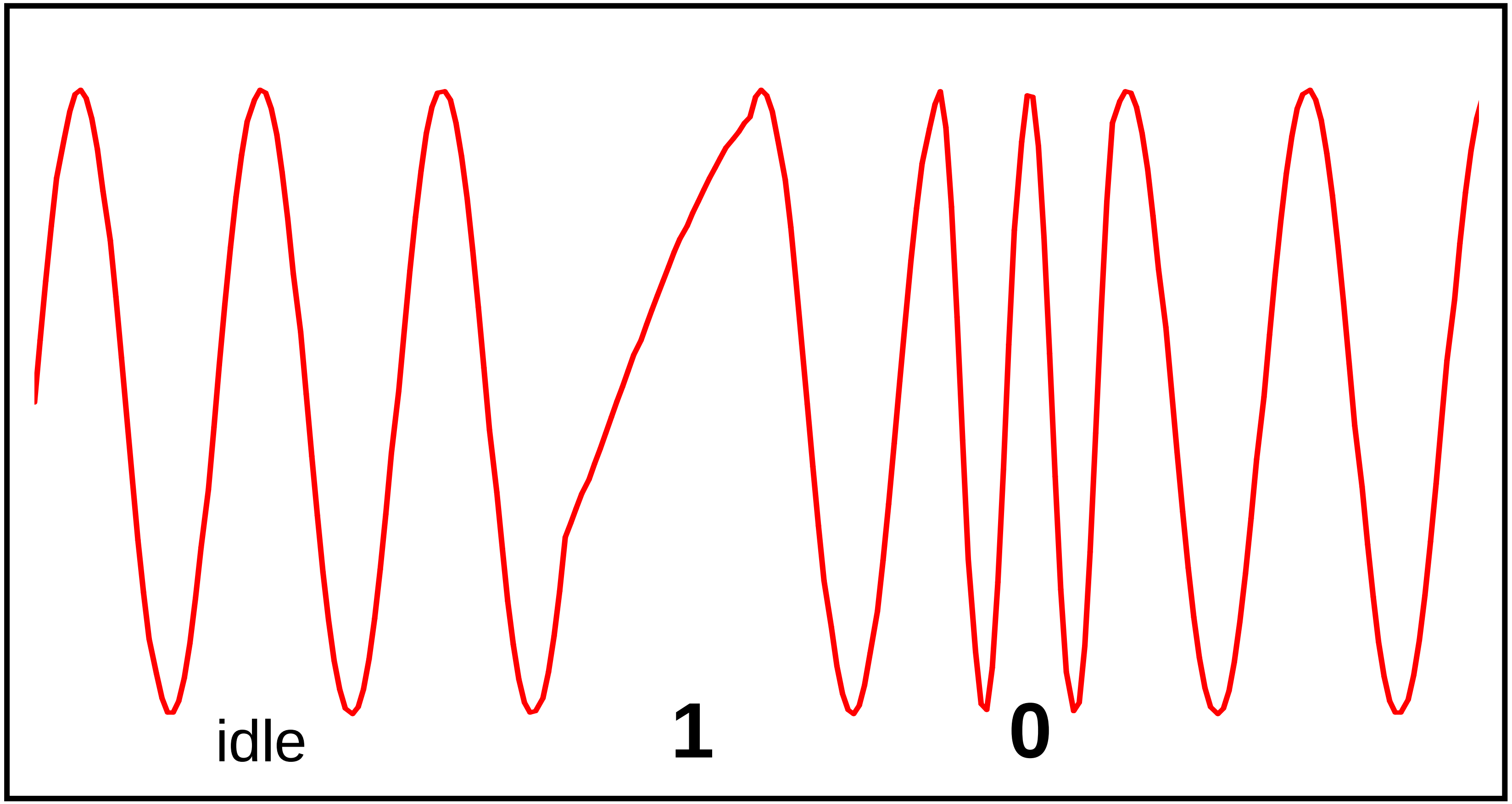
- Original signal
- Amplitude modulation
- Frequency modulation



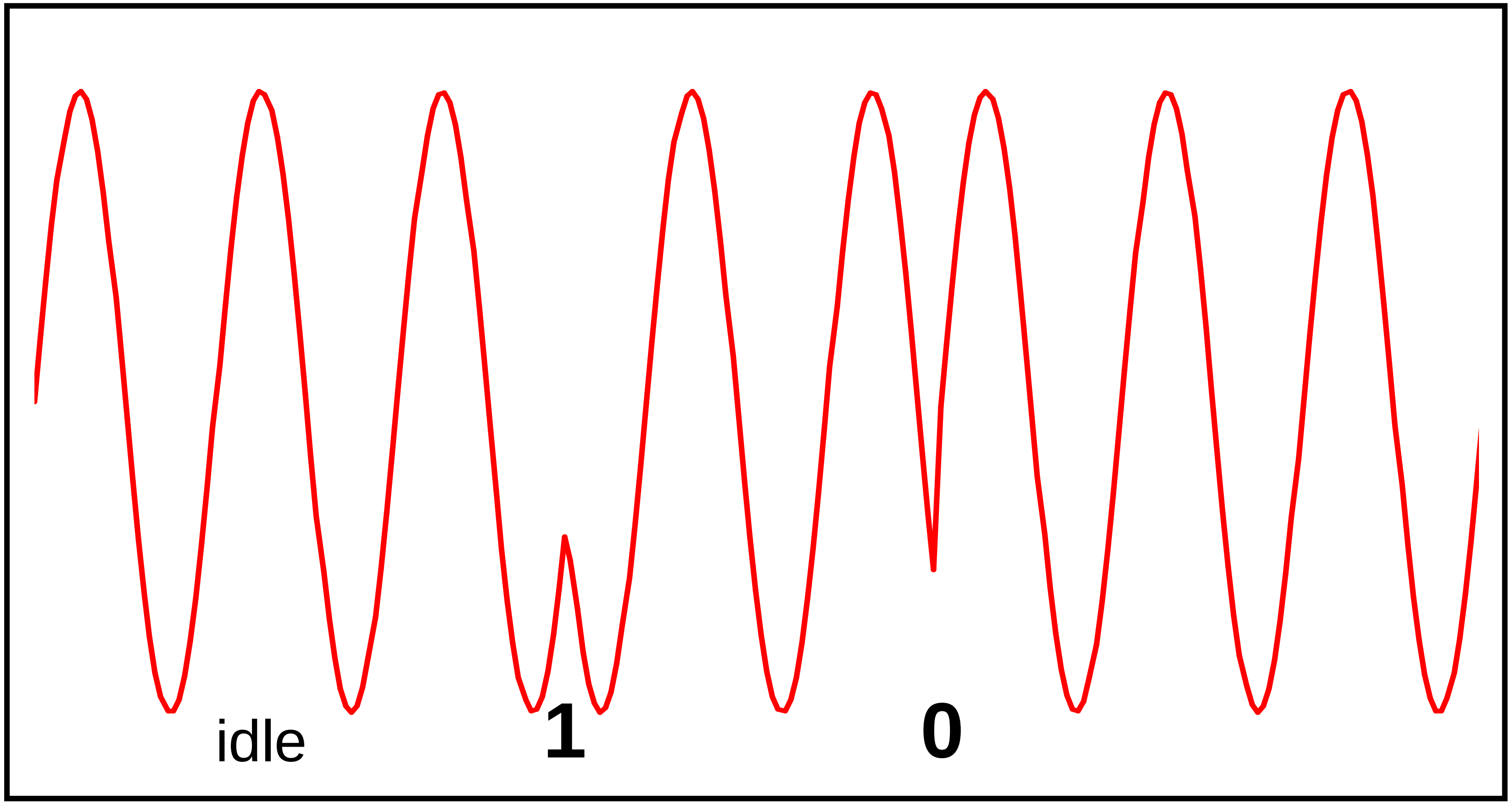
Amplitude Modulation



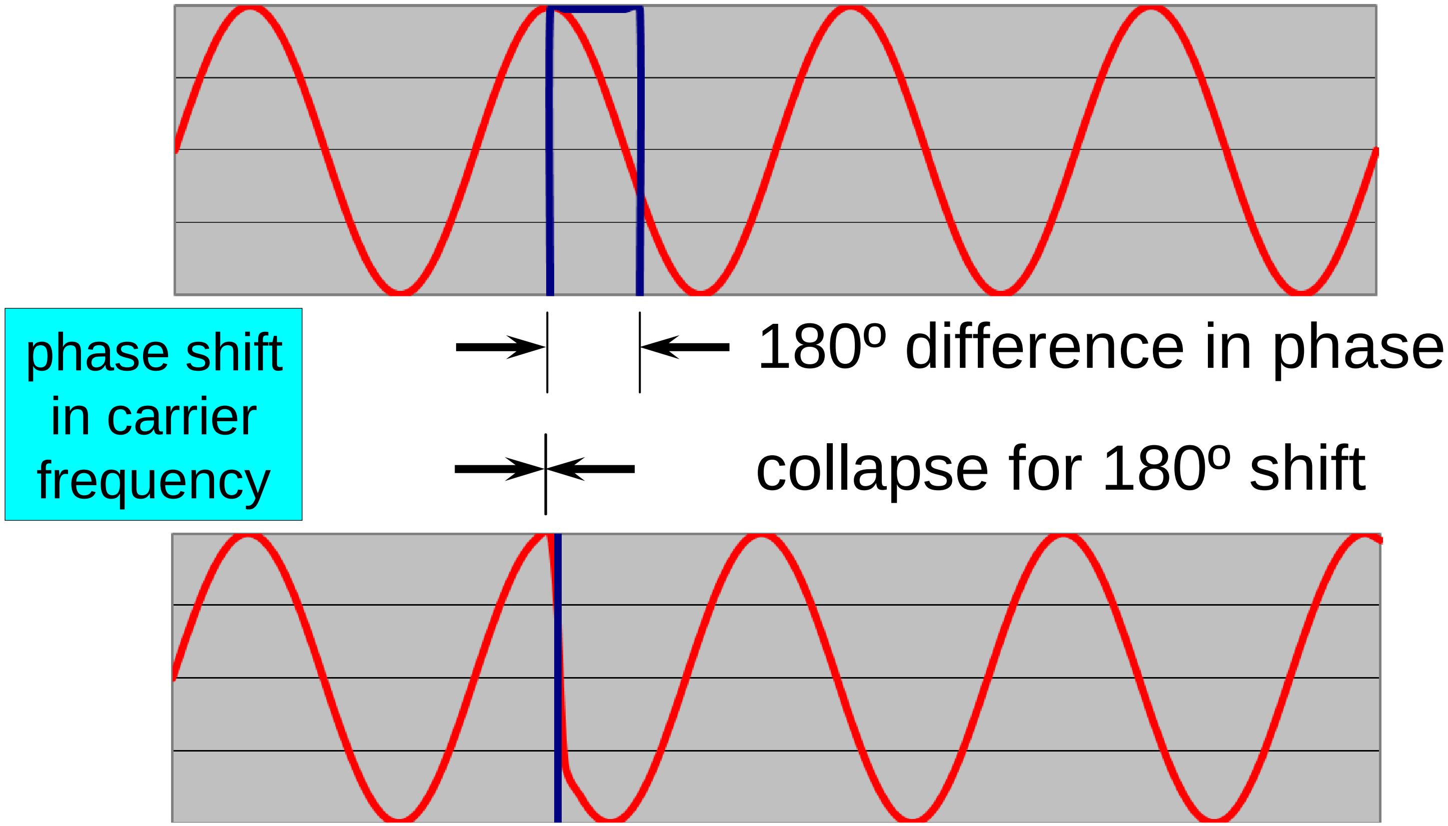
Frequency Modulation



Phase Modulation



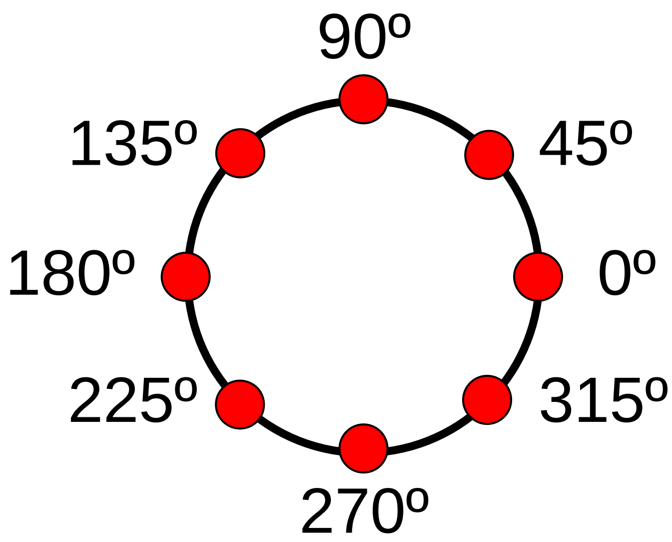
Phase Modulation



Phase Modulation Algorithm

- Send carrier frequency for one period
  - Perform phase shift
  - Shift value encodes symbol
    - Value in range [0, 360º)
    - Multiple values for multiple symbols
    - Represent as circle

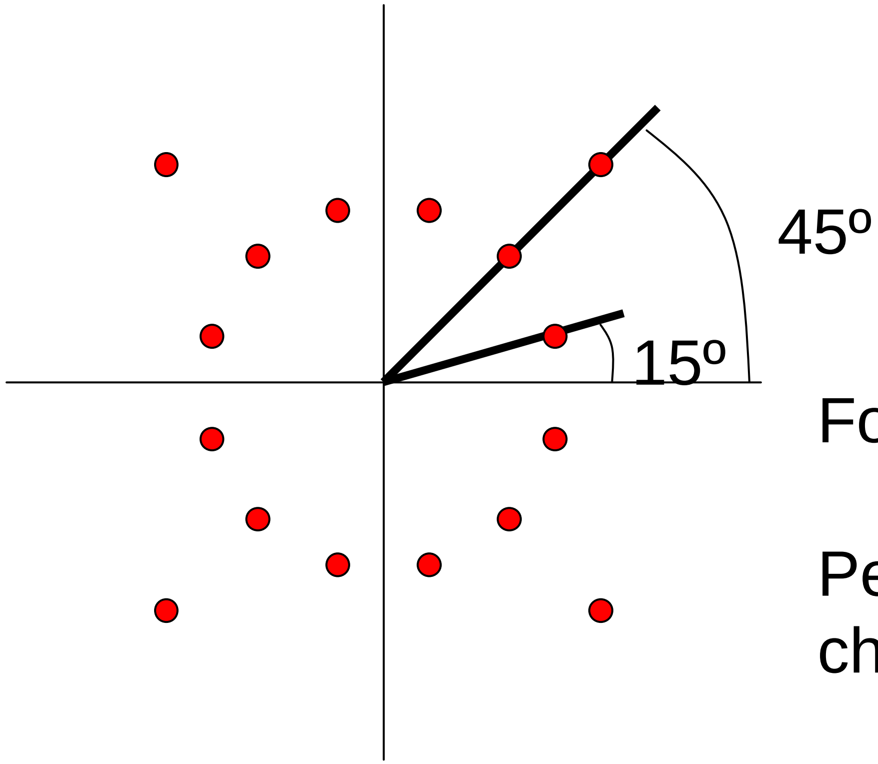
8-symbol example



V.32 9600 bps

- Communication between modems
- Analog phone line
- Uses a combination of amplitude and phase modulation
  - Known as Quadrature Amplitude Modulation (QAM)
- Sends one of 16 signals each clock cycle

Constellation Pattern for V.32 QAM



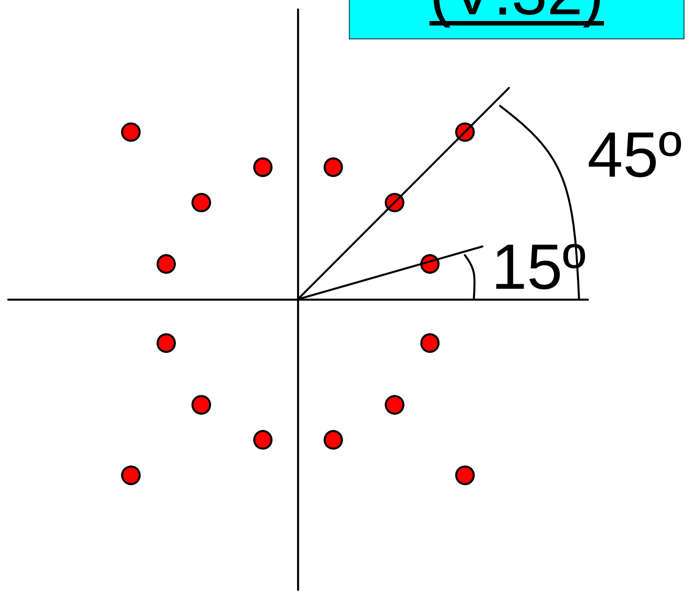
For a given symbol:

Perform phase shift and change to new amplitude

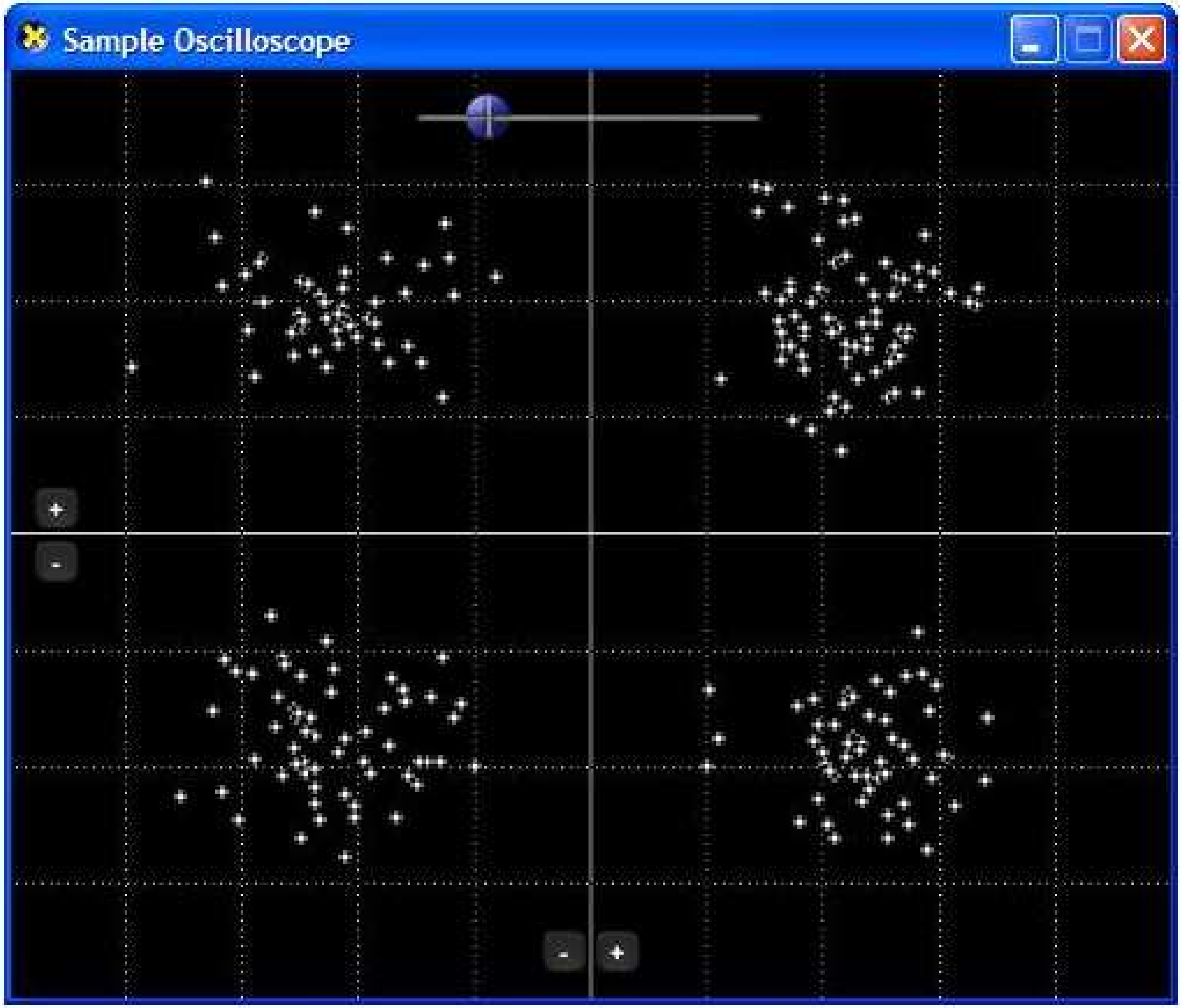
Quadrature Amplitude Modulation (QAM)

- Same algorithm as phase modulation
- Can also change signal amplitude
- 2-dimensional representation
  - Angle is phase shift
  - Radial distance is new amplitude

16-symbol example (V.32)



Example constellation



Comments on V.32

- V.32 transmits at 2400 baud
  - *i.e.*, 2,400 symbols per second
- Each symbol contains
  - $\log_2 16 = 4$ bits
- Data rate
  - 4 x 2400 = 9600 bps
- Points in constellation diagram
  - Chosen to maximize error detection
  - Process called trellis coding

Generalizing the Examples

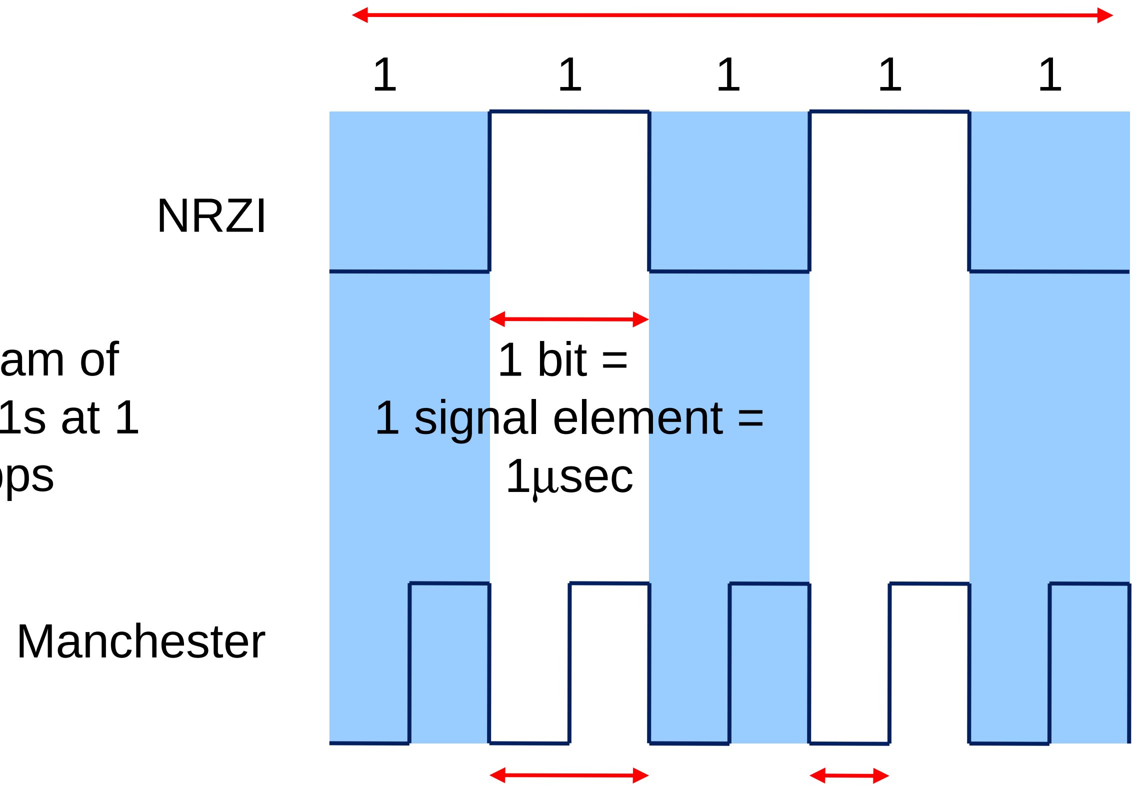
- What limits baud rate?
- What data rate can a channel sustain?
- How is data rate related to bandwidth?
- How does noise affect these bounds?
- What else can limit maximum data rate?

What Limits Baud Rate?

- Baud rate
  - Typically limited by electrical signaling properties
- Changing voltages takes time
  - No matter how small the voltage or how short the wire
- Electronics
  - Slow compared to optics
- Note
  - Baud rate can be as high as twice the frequency (bandwidth) of communication
  - One cycle can contain two symbols

Modulation Rate

5 bits = 5µsec



A stream of binary 1s at 1 Mbps

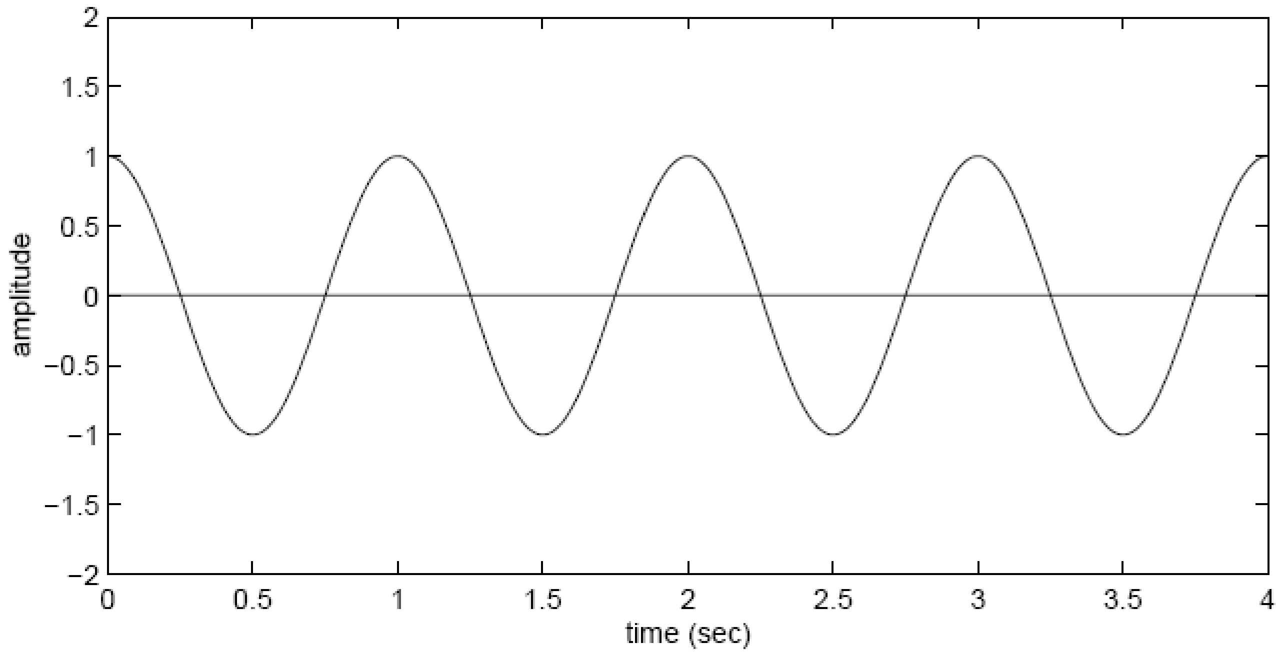
1 bit = 1µsec

1 signal element = 0.5µsec

Data Rate (R)
= bits/sec
= 1 Mbps for both

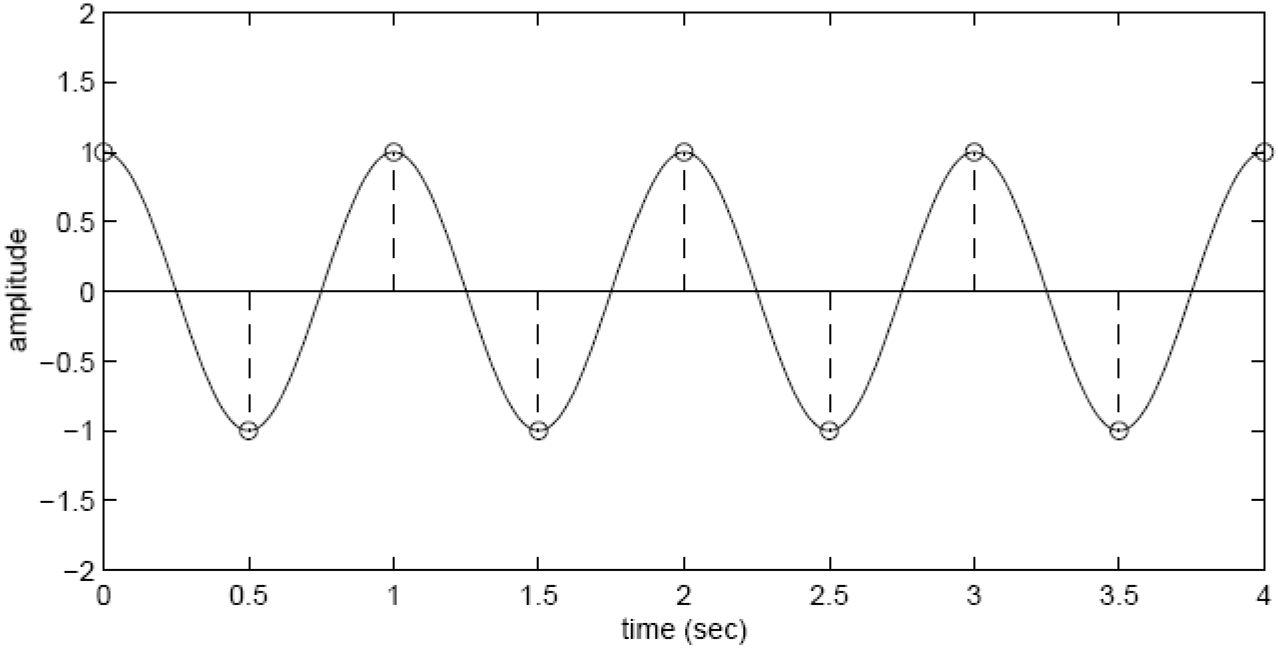
Modulation Rate
= Baud Rate
= Rate at which signal elements are generated
= R (NRZI)
= 2R (Manchester)

Sampling



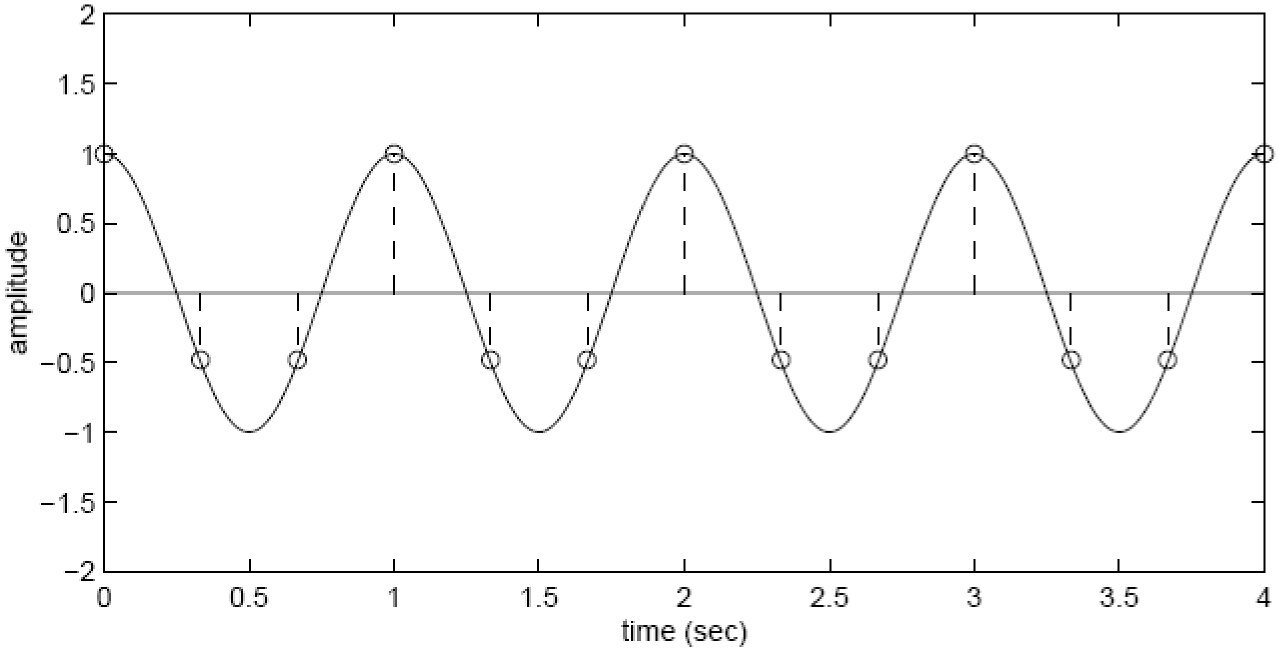
- Suppose you have the following 1Hz signal being received
- How fast to sample, to capture the signal?

Sampling



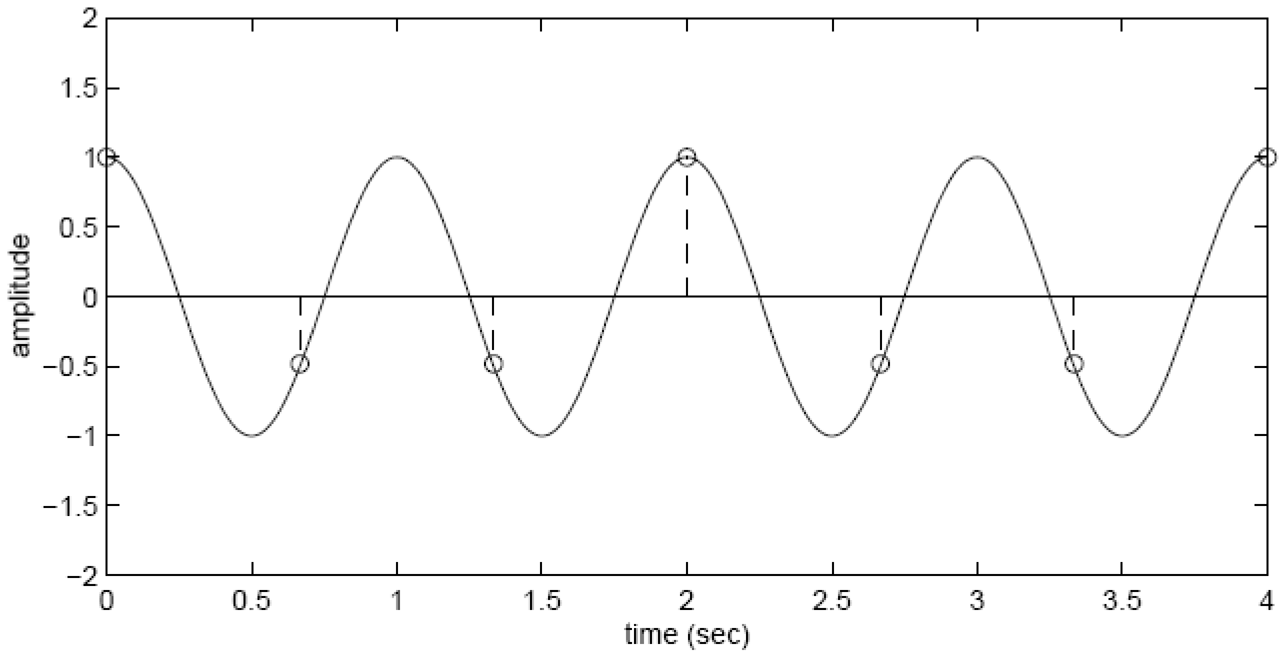
- Sampling a 1 Hz signal at 2 Hz is enough
  - Captures every peak and trough

Sampling

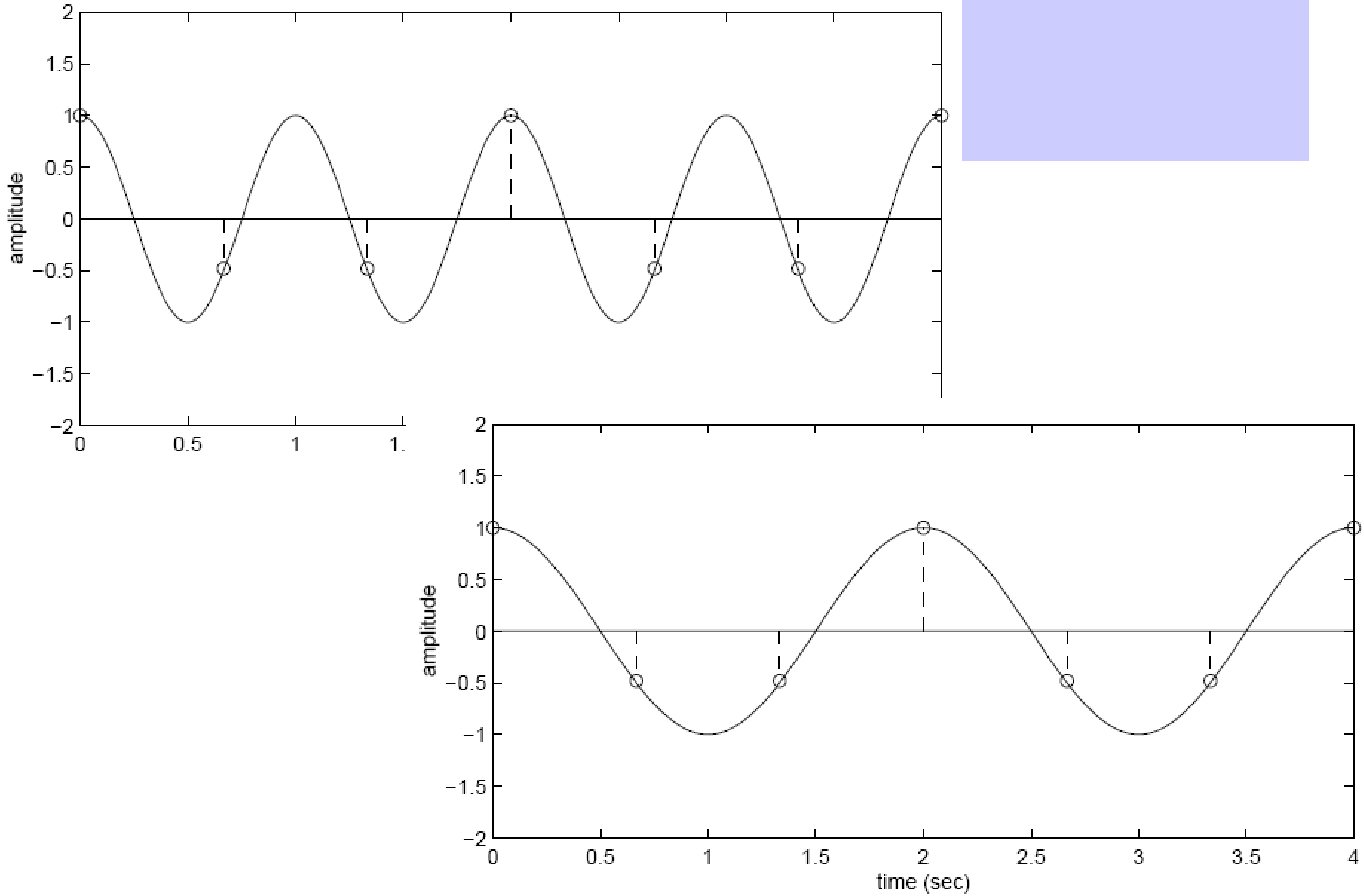


- Sampling a 1 Hz signal at 3 Hz is also enough
  - In fact, more than enough samples to capture variation in signal

Sampling

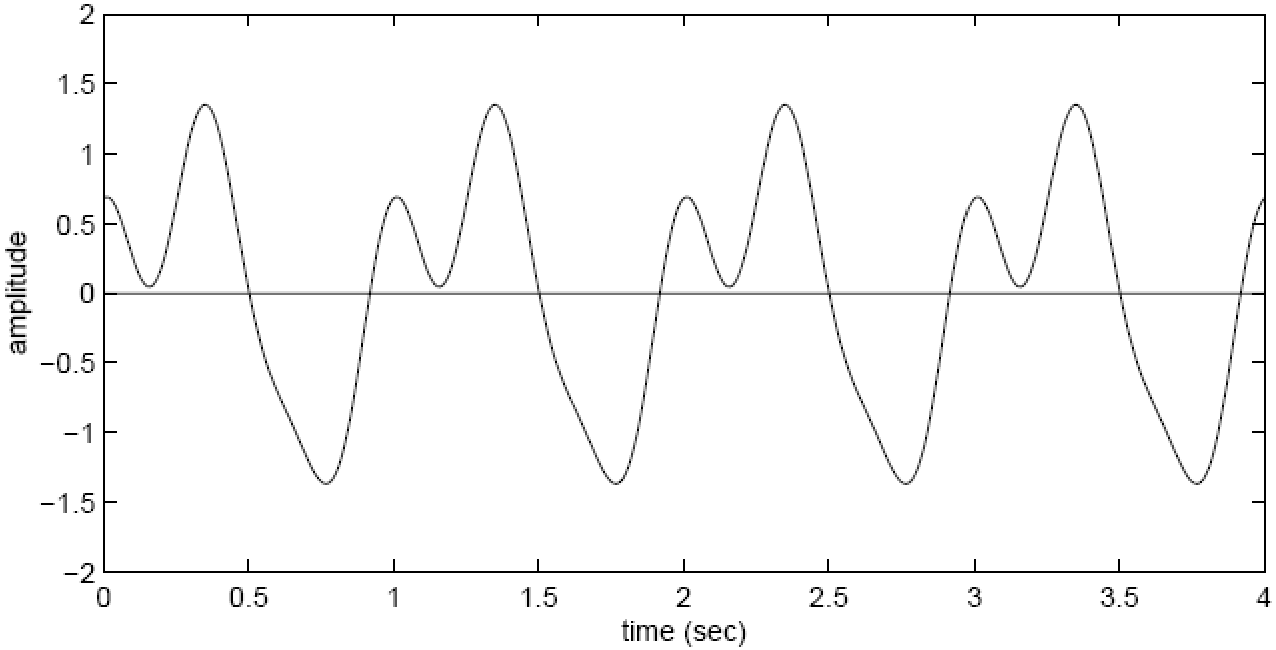


- Sampling a 1 Hz signal at 1.5 Hz is not enough
  - Why?



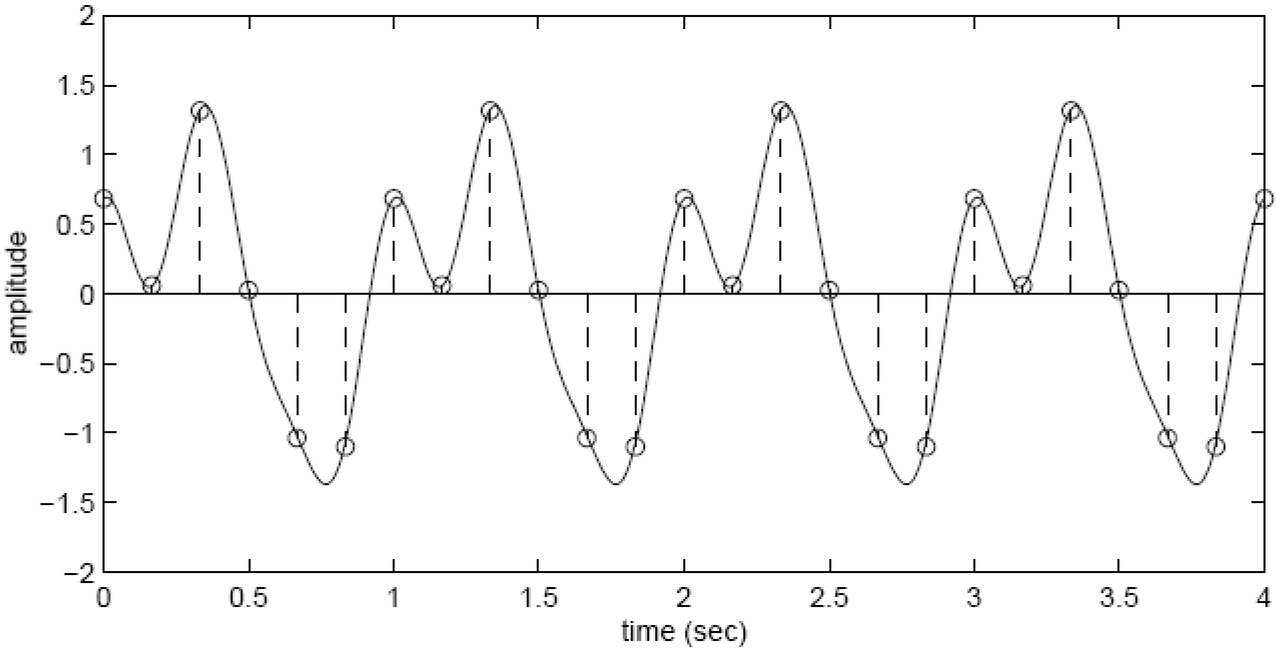
- Sampling a 1 Hz signal at 1.5 Hz is not enough
  - Not enough samples, can’t distinguish between multiple possible signals

What about more complex signals?



- Fourier's theorem: any continuous signal can be decomposed into a sum of sines and cosines at different frequencies
- Example: Sum of 1 Hz, 2 Hz, and 3 Hz sines
  - How fast to sample?

What about more complex signals?



- Fourier's theorem: any continuous signal can be decomposed into a sum of sines and cosines at different frequencies
- Example: Sum of 1 Hz, 2 Hz, and 3 Hz sines
  - How fast to sample? **--> answer: 6 Hz**

What Data Rate can a Channel Sustain? How is Data Rate Related to Bandwidth?

- Transmitting N distinct signals over a noiseless channel with bandwidth B, we can achieve at most a data rate of

$$2B \log_2 N$$

- Nyquist's Sampling Theorem (H. Nyquist, 1920's)
  - Sampling rate = 2B
  - A higher sampling rate is pointless because higher frequency signals have been filtered out

Noiseless Capacity

- Nyquist's theorem: $2B \log_2 N$
- Example 1: sampling rate of a phone line
  - B = 4000 Hz
  - 2B = 8000 samples/sec.
    - sample every 125 microseconds
- Example 2: noiseless capacity
  - B = 1200 Hz
  - N = each pulse encodes 16 levels
  - C = $2B \log_2 (N) = D \times \log_2 (N)$

    $= 2400 \times 4 = 9600$ bps.

What can Limit Maximum Data Rate?

- Noise
  - E.g., thermal noise (in-band noise) can blur symbols
- Transitions between symbols
  - Introduce high-frequency components into the transmitted signal
  - Such components cannot be recovered (by Nyquist's Theorem), and some information is lost
- Examples
  - Phase modulation
    - Single frequency (with different phases) for each symbol
    - Transitions can require very high frequencies

How does Noise affect these Bounds?

- In-band (thermal, not high-frequency) noise
  - Blurs the symbols, reducing the number of symbols that can be reliably distinguished.
- Claude Shannon (1948)
  - Extended Nyquist's work to channels with additive white Gaussian noise (a good model for thermal noise)

$$\text{channel capacity } C = B \log_2 (1 + S/N)$$

B is the channel bandwidth

S/N is the ratio between

the average signal power and

the average in-band noise power

Noisy Capacity

$$\mathrm{SNR(dB)} = 10\log_{10}\left(\frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}\right)$$

- Telephone channel
  - 3400 Hz at 40 dB SNR
  - $C = B \log_2 (1+S/N)$ bits/s
  - SNR = 40 dB

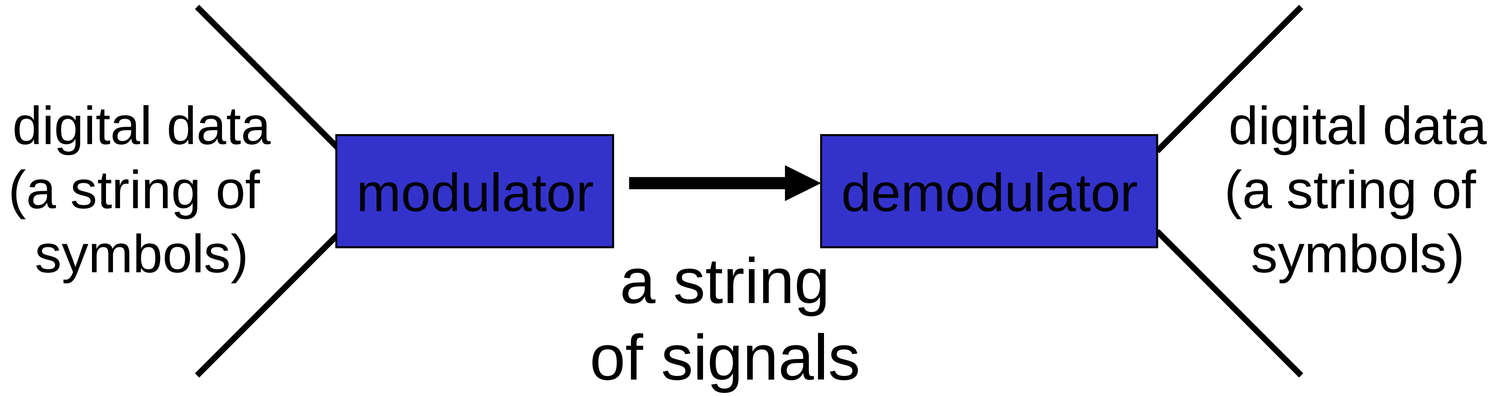
    $40 = 10 \log_{10} (S/N)$

    $S/N = 10{,}000$
  - $C = 3400 \log_2 (10001) = 44.8$ kbps

Summary of Encoding

- Problems
  - Attenuation, dispersion, noise
- Digital transmission allows periodic regeneration
- Variety of binary voltage encodings
  - High frequency components limit to short range
  - More voltage levels provide higher data rate
- Carrier frequency and modulation
  - Amplitude, frequency, phase, and combinations
  - Quadrature amplitude modulation: amplitude and phase, many signals
- Nyquist (noiseless) and Shannon (noisy) limits on data rates

Framing



- Encoding translates symbols to signals
- Framing demarcates units of transfer
  - Separates continuous stream of bits into frames
  - Marks start and end of each frame

Framing

- Demarcates units of transfer
- Goal
  - Enable nodes to exchange blocks of data
- Challenge
  - How can we determine exactly what set of bits constitute a frame?
  - How do we determine the beginning and end of a frame?

Benefits of framing

- Synchronization recovery
  - Breaks up continuous streams of unframed bytes
  - Recall RS-232 start and stop bits
- Link multiplexing
  - Multiple hosts on shared medium
  - Simplifies multiplexing of logical channels
- Efficient error detection
  - Per-frame error checking and recovery

Framing

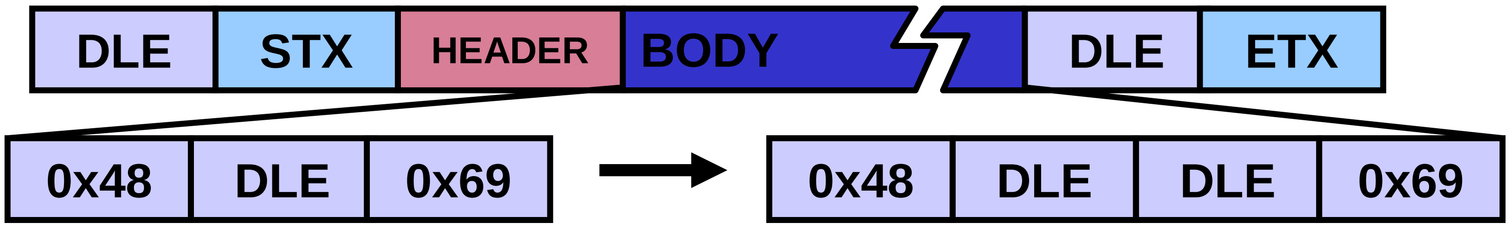
- Approaches
  - Sentinel: delimiter at end of frame
  - Length-based: length field in header
  - Clock based: periodic, time-based
- Characteristics
  - Bit- or byte-oriented
  - Fixed or variable length
  - Data-dependent or data-independent length

Sentinel-Based Framing

- End of Frame
  - Marked with a special byte or bit pattern
    - Frame length is data-dependent
  - Challenge
    - Frame marker may exist in data
    - Requires stuffing
- Examples
  - BISYNC, HDLC, PPP, IEEE 802.4 (token bus)

ARPANET IMP-IMP

- Interface Message processors (IMPs)
  - Packet switching nodes in the original ARPANET
  - Byte oriented, Variable length, Data dependent
  - Frame marker bytes
    - STX/ETX start of text/end of text
    - DLE data link escape
  - Byte Stuffing
    - DLE byte in data sent as two DLE bytes back-to-back



High-Level Data Link Control Protocol (HDLC)

- Bit oriented, Variable length, Data-dependent
- Frame Marker
  - 01111110
- Bit Stuffing
  - Insert 0 after pattern 011111 in data
  - Example
    - 01111110 end of frame
    - 01111111 error! lose one or two frames

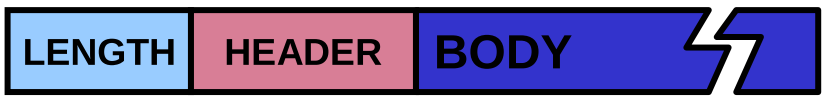
- *Handout: problem 2*

IEEE 802.4 (token bus)

- Alternative to Ethernet (802.3) with fairer arbitration
- End of frame marked by encoding violation,
  - i.e., physical signal not used by valid data symbol
  - Recall Manchester encoding
    - low-high means "0"
    - high-low means "1"
    - low-low and high-high are invalid
- IEEE 802.4
  - byte-oriented, variable-length, data-independent
- Another example
  - Fiber Distributed Data Interface (FDDI) uses 4B/5B
- Technique also applicable to bit-oriented framing

Length-Based Framing

- End of frame
  - Calculated from length sent at start of frame
  - Challenge
    - Corrupt length markers
- Examples
  - DECNET's DDCMP
    - Byte-oriented, variable-length
  - RS-232 framing
    - Bit-oriented, implicit fixed-length

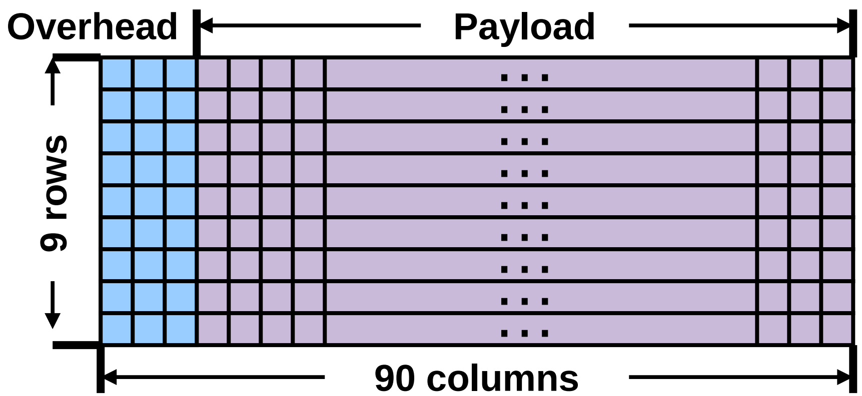


Clock-Based Framing

- Continuous stream of fixed-length frames
  - Clocks must remain synchronized
- STS-1 frames - 125μs long
  - No bit or byte stuffing
- Example
  - Synchronous Optical Network (SONET)
- Problems
  - Frame synchronization
  - Clock synchronization

SONET

- Frames (all STS formats) are 125 µsec long
  - Ex: STS-1 – 51.84 Mbps = 90 bytes
- Frame Synchronization
  - 2-byte synchronization pattern at start of each frame

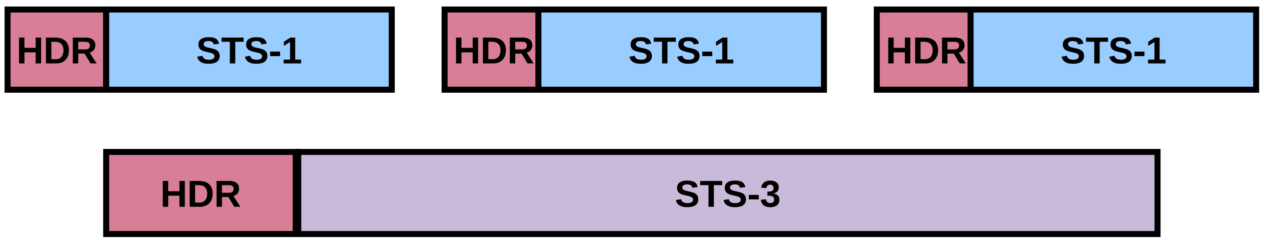


SONET: Challenges

- How to recover frame synchronization
  - Synchronization pattern unlikely to occur in data
    - Wait until pattern appears in same place repeatedly
- How to maintain clock synchronization
  - NRZ encoding
    - Data scrambled (XOR'd) with 127-bit pattern
    - Creates transitions
    - Also reduces chance of finding false sync. pattern

SONET

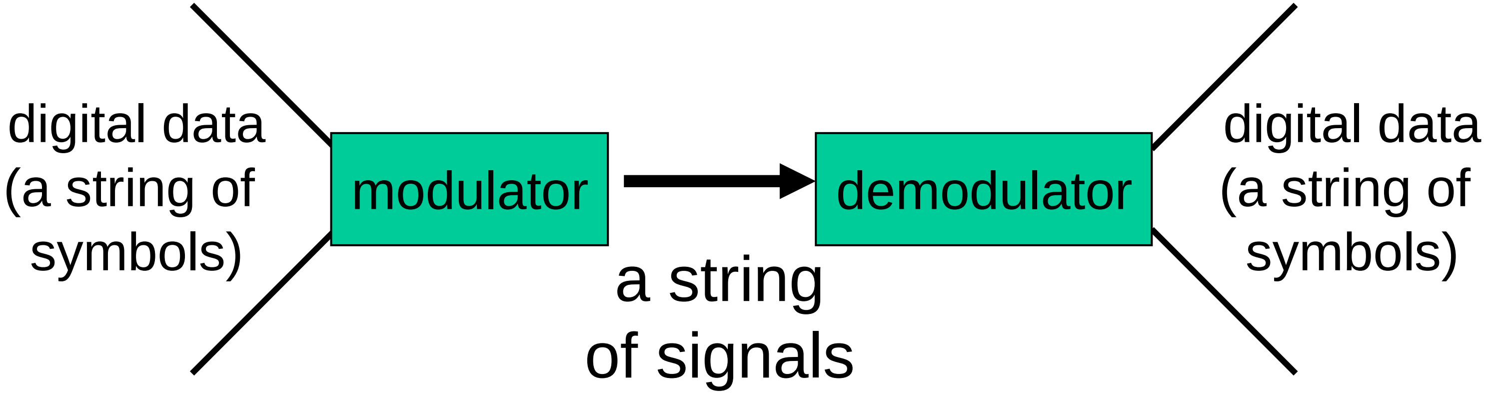
- A single SONET frame may contain multiple smaller SONET frames
- Bytes from multiple SONET frames are interleaved to ensure pacing



Framing Summary

- Technique
  - Demarcate units of transfer
- Benefits
  - Synchronization recovery
  - Link multiplexing
  - Efficient error detection
- Approaches
  - Sentinel
  - Length-based Clock based
- Characteristics
  - Bit- or byte-oriented
  - Fixed or variable length
  - Data-dependent or data-independent length

Error Detection



- **Encoding** translates symbols to signals
- **Framing** demarcates units of transfer
- **Error detection** validates correctness of each frame

Error Detection

- Idea
  - Add redundant information that can be used to determine if errors have been introduced, and potentially fix them

- Errors checked at many levels
  - Demodulation of signals into symbols (analog)
  - Bit error detection/correction (digital)—our main focus
    - Within network adapter (CRC check)
    - Within IP layer (IP checksum)
    - Possibly within application as well

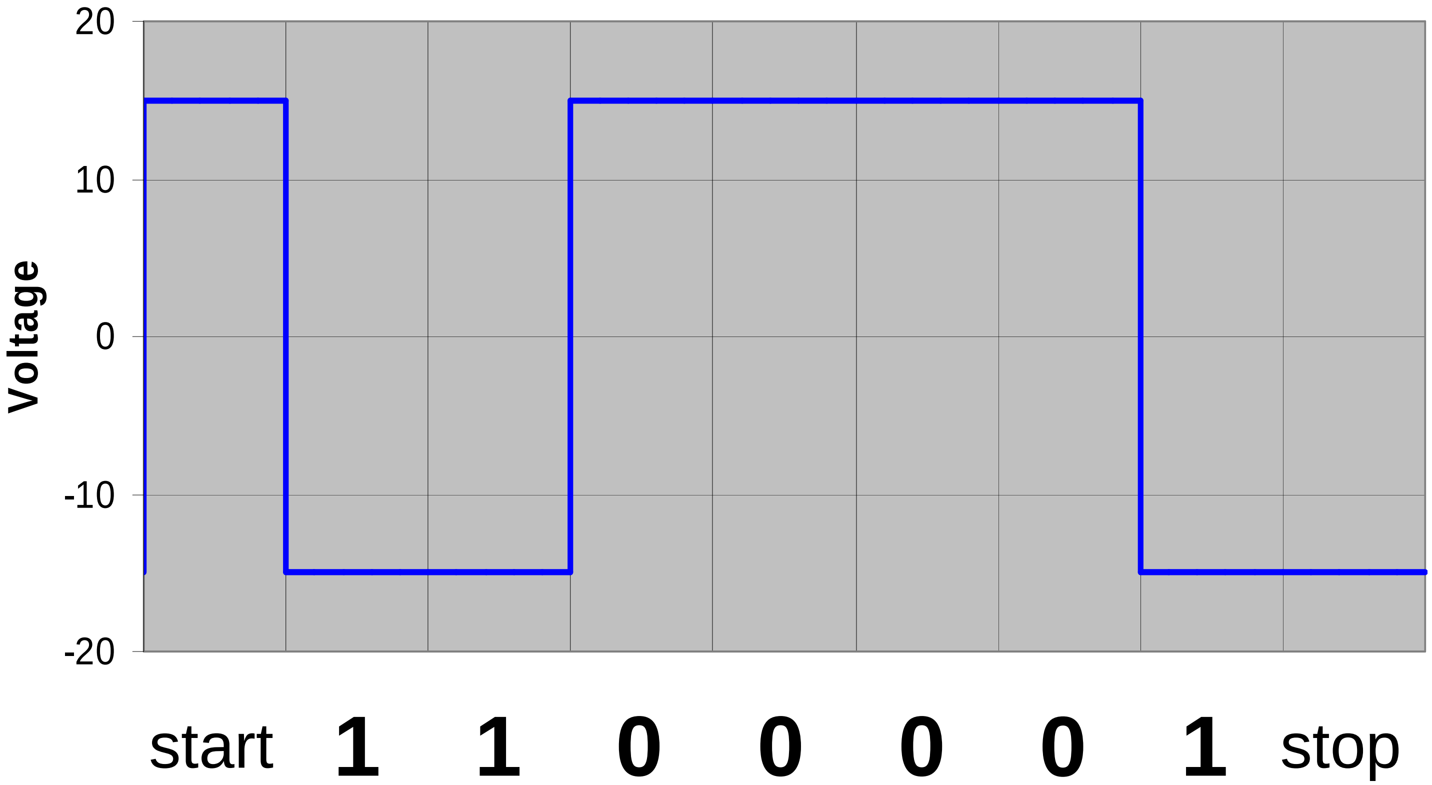
Error Detection

- Analog Errors
  - Example of signal distortion
- Hamming distance
  - Parity and voting
  - Hamming codes
- Error bits or error bursts?
- Digital error detection
  - Two-dimensional parity
  - Checksums
  - Cyclic Redundancy Check (CRC)

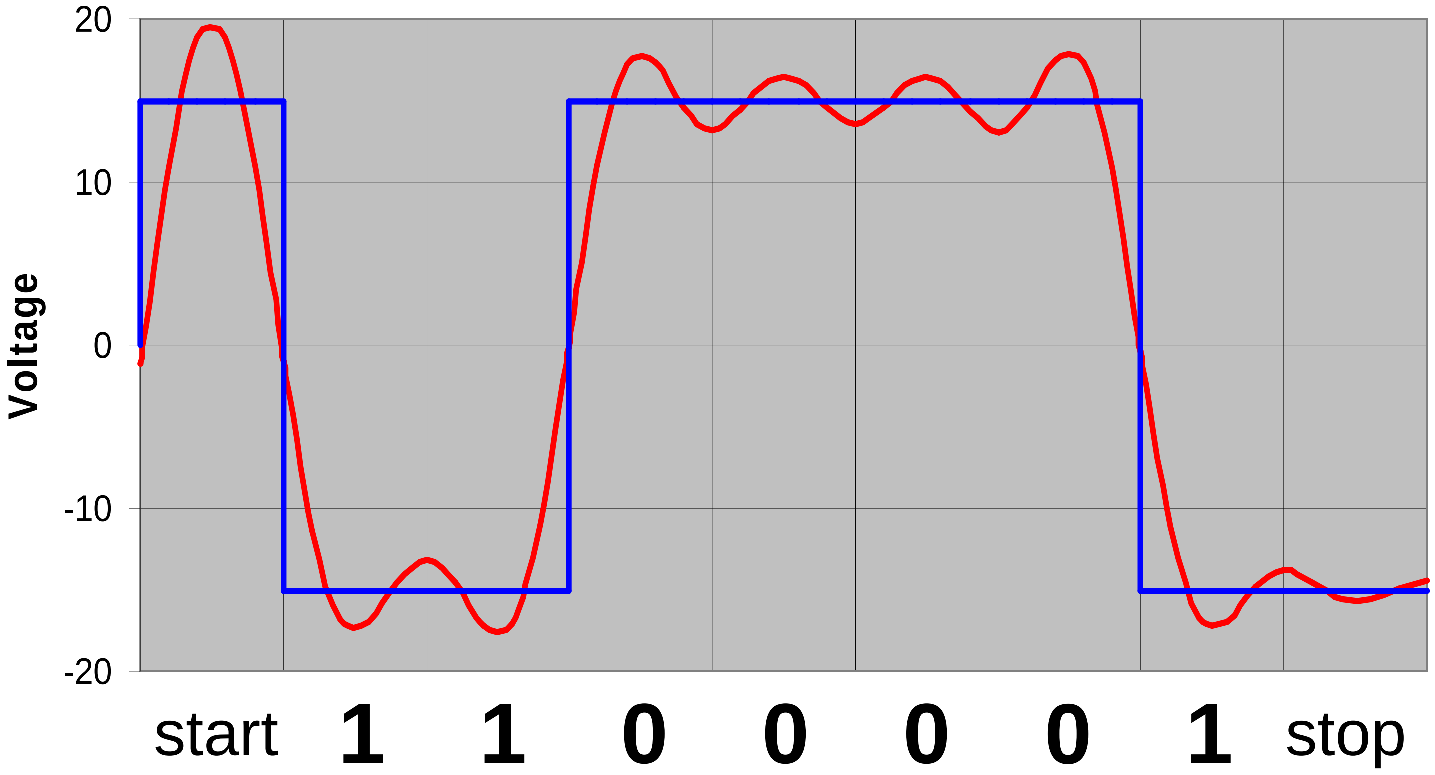
Analog Errors

- Consider RS-232 encoding of character ‘Q’
  – ASCII Q = 1100001
- Assume idle wire (-15V) before and after signal

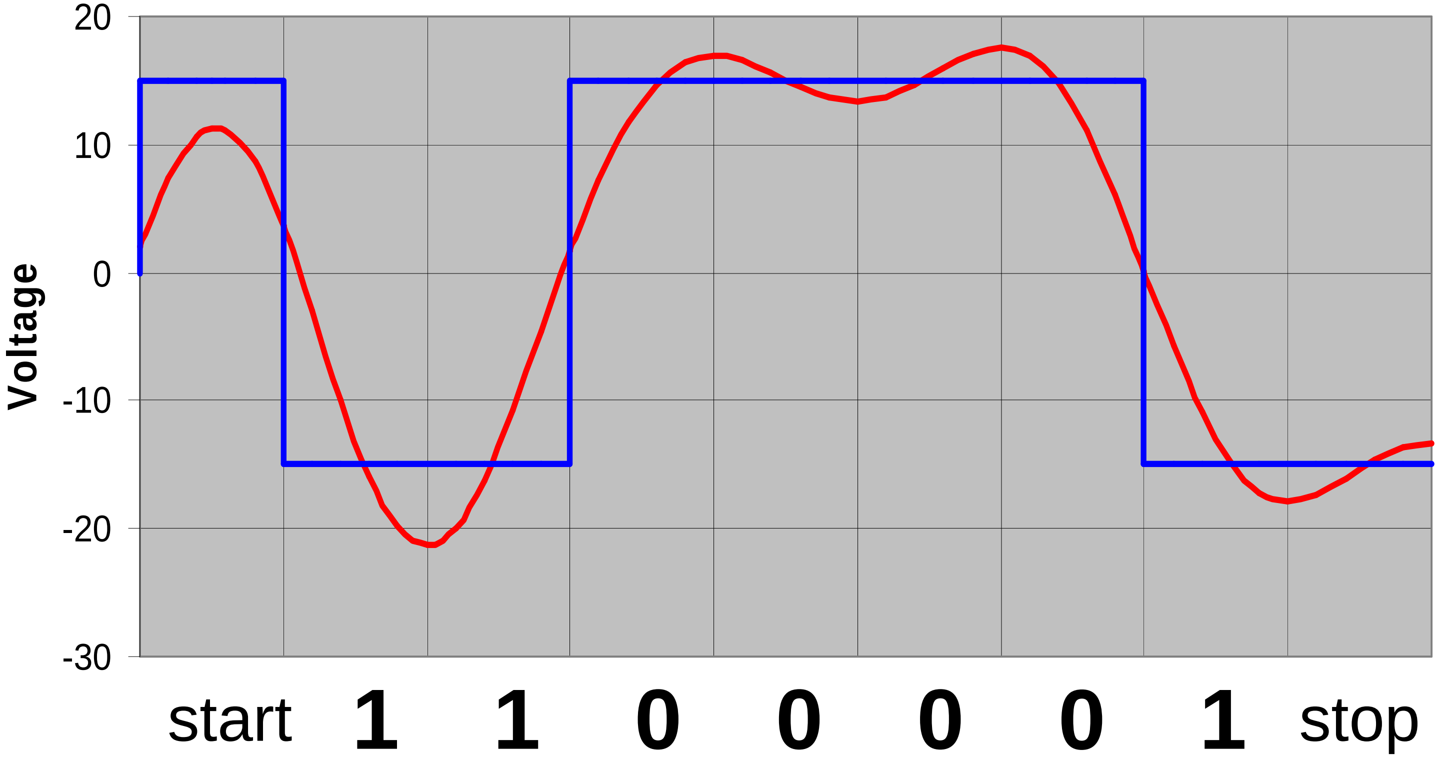
RS-232 Encoding of 'Q'



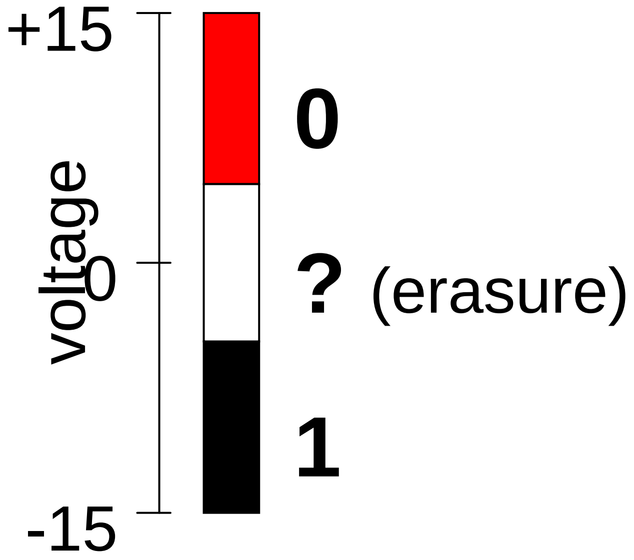
Limited-Frequency Signal Response (bandwidth = baud rate)



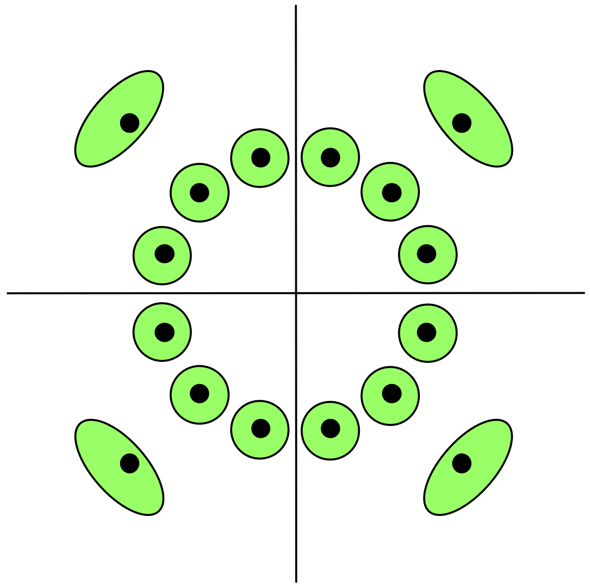
Limited-Frequency Signal Response (bandwidth = baud rate/2)



Symbols



possible binary voltage encoding symbol neighborhoods and erasure region



possible QAM symbol neighborhoods in green; all other space results in erasure

Symbols

- Inputs to digital level
  - valid symbols
  - erasures
- Hamming distance
  - Definition
  - 1-bit error-detection with parity
  - 1-bit error-correction with voting
  - 2-bit erasure-correction with voting
  - Hamming codes (1-bit error correction)

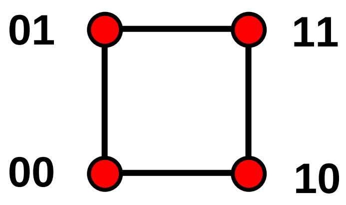
Hamming Distance

- The Hamming distance between two code words is the minimum number of bit flips to move from one to the other
  - Example:
  - 00101 and 00010
  - Hamming distance of 3

Detecting **bit flips with** ***Parity***

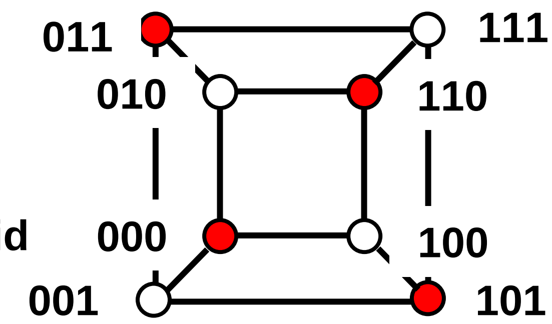
- 1-bit error detection with parity
  - Add an extra bit to a code to ensure an even (odd) number of 1s
  - Every code word has an even (odd) number of 1s

Valid code words



Parity Encoding:

White – invalid (error)



Correcting bit flips with *Voting*

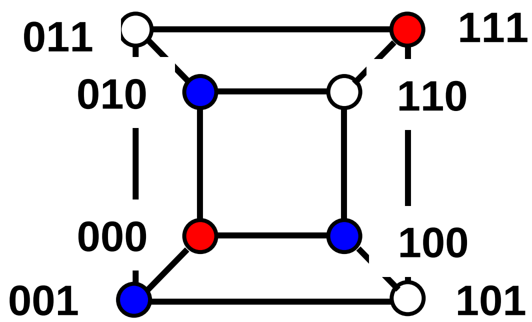
- 1-bit error correction with voting
  - Every codeword is transmitted n times

Valid code words

Voting:

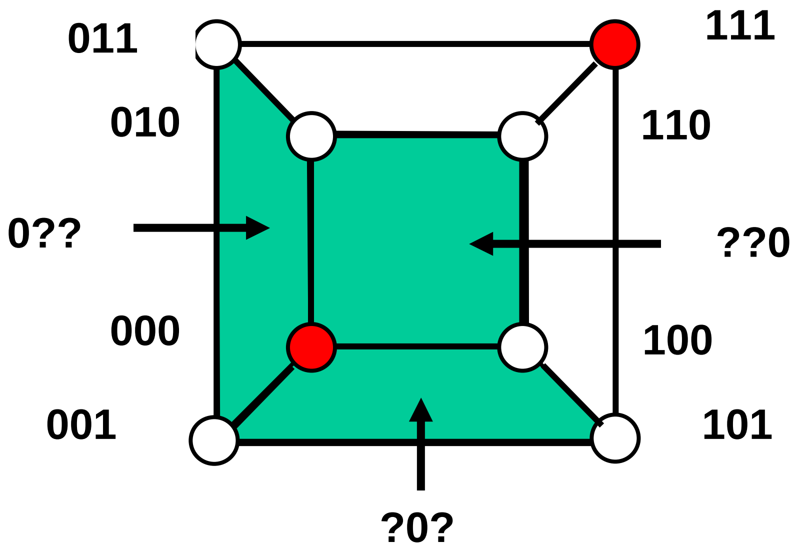
White – correct to 1

Blue - correct to 0



2-bit Erasure Correction with Voting

- Every code word is copied 3 times



2-erasure planes in green remaining bit not ambiguous

cannot correct 1-error and 1-erasure

Minimum Hamming Distance

- The minimum Hamming distance of a code is the minimum distance over all pairs of codewords
  - Minimum Hamming Distance for parity
    - 2
  - Minimum Hamming Distance for voting
    - 3

Coverage

- N-bit error detection
  - No code word changed into another code word
  - Requires Hamming distance of N+1
- N-bit error correction
  - N-bit neighborhood: all codewords within N bit flips
  - No overlap between N-bit neighborhoods
  - Requires hamming distance of 2N+1

Hamming Codes

- Linear error-correcting code, Named after Richard Hamming
  - Simple, commonly used in RAM (e.g., ECC-RAM)
- Can detect up to 2 simultaneous bit errors
- Can correct single-bit errors
- Construction
  - number bits from 1 upward
  - powers of 2 are check bits
  - all others are data bits
  - Check bit j is XOR of all bits k such that (j AND k) = j
- Example: 4 bits of data, 3 check bits

1	2	3	4	5	6	7	8	
C	C	D	C	D	D	D	C	…

Hamming Codes

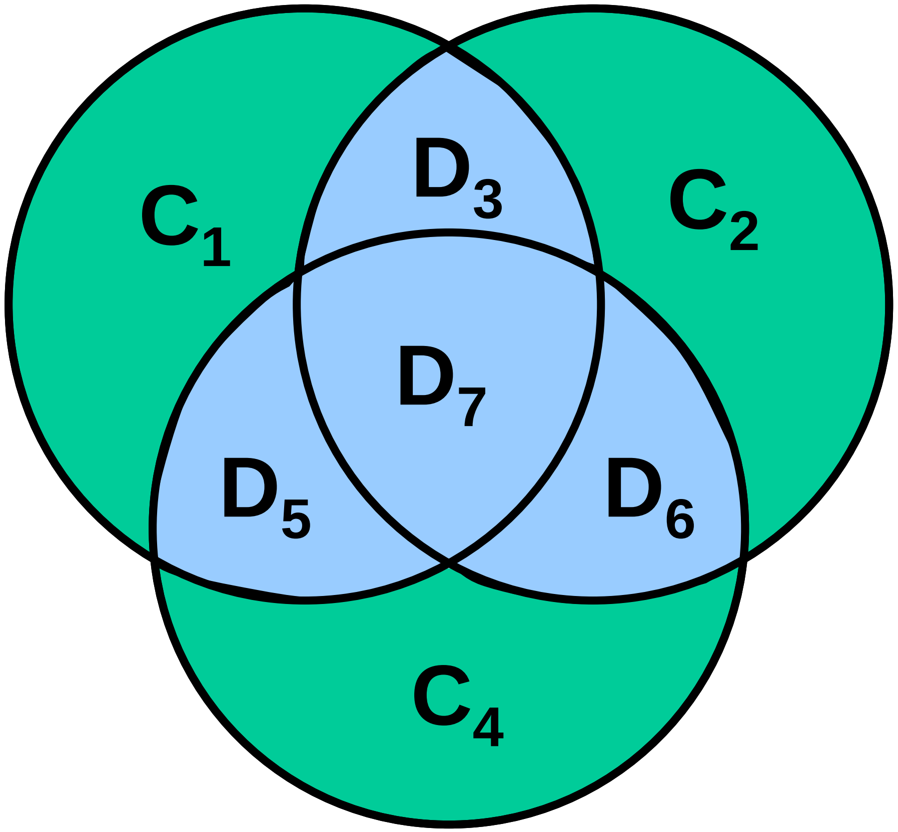
1	2	3	4	5	6	7
C_1	C_2	D_3	C_4	D_5	D_6	D_7

C1 = D3 XOR D5 XOR D7

C2 = D3 XOR D6 XOR D7

C4 = D5 XOR D6 XOR D7

Hamming Codes



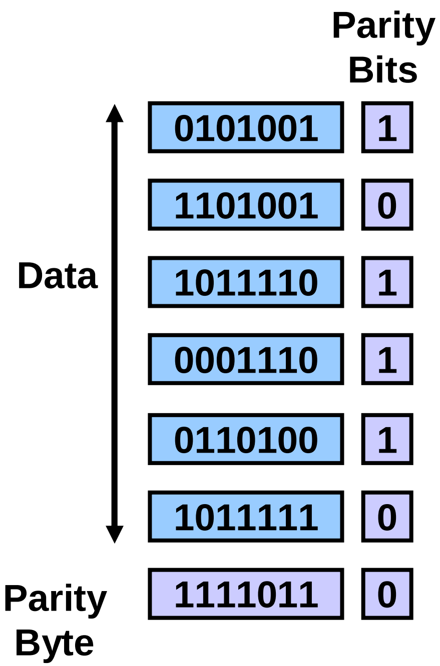
Error Bits or Bursts?

- Common model of errors
  - Probability of error per bit
  - Error in each bit independent of others
  - Value of incorrect bit independent of others
- Burst model
  - Probability of back-to-back bit errors
  - Error probability dependent on adjacent bits
  - Value of errors may have structure
- Why assume bursts?
  - Appropriate for some media (e.g., radio)
  - Faster signaling rate enhances such phenomena

Digital Error Detection Techniques

- Two-dimensional parity
  - Detects up to 3-bit errors
  - Good for burst errors
- IP checksum
  - Simple addition
  - Simple in software
  - Used as backup to CRC
- Cyclic Redundancy Check (CRC)
  - Powerful mathematics
  - Tricky in software, simple in hardware
  - Used in network adapter

Two-Dimensional Parity



- Use 1-dimensional parity
  - Add one bit to a 7-bit code to ensure an even/odd number of 1s
- Add 2nd dimension
  - Add an extra byte to frame
    - Bits are set to ensure even/odd number of 1s in that position across all bytes in frame
- Comments
  - Catches all 1-, 2- and 3-bit and most 4-bit errors

Two-Dimensional Parity

0	1	0	0	0	1	1	1	0
0	1	1	0	0	1	0	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	0	1	0	0	1
0	0	1	0	0	0	1	1	1

Internet Checksum

- Idea: Add up all the words, transmit the sum
- Internet Checksum
  - Use 1's complement addition on 16bit codewords
  - Example
    - Codewords: -5 -3
    - 1's complement binary: 1010 1100
    - 1's complement sum 1000
- Comments
  - Small number of redundant bits
  - Easy to implement
  - Not very robust

IP Checksum

```
u_short cksum(u_short *buf, int count) {
    register u_long sum = 0;
    while (count--) {
        sum += *buf++;
        if (sum & 0xFFFF0000) {
        /* carry occurred, so wrap around */
                sum &= 0xFFFF;
                sum++;
        }
    }
    return ~(sum & 0xFFFF);
}
```

Cyclic Redundancy Check (CRC)

- Non-secure hash function based on cyclic codes
- Idea
  - Add **k** bits of redundant data to an **n**-bit message
  - **N**-bit message is represented as a **n**-degree polynomial with each bit in the message being the corresponding coefficient in the polynomial
  - Example
    - Message = 10011010
    - Polynomial

$$= \mathbf{1} * x^7 + \mathbf{0} * x^6 + \mathbf{0} * x^5 + \mathbf{1} * x^4 + \mathbf{1} * x^3 + \mathbf{0} * x^2 + \mathbf{1} * x + \mathbf{0}$$

$$= x^7 + x^4 + x^3 + x$$

CRC Approach

- Given
  - Message M(x) 10011010
  - Represented as $x^7 + x^4 + x^3 + x$

1. Select a divisor polynomial C(x) with degree k
   - Example with k = 3:
     - $C(x) = x^3 + x^2 + 1$
     - Represented as 1101
2. Transmit a polynomial P(x) that is evenly divisible by C(x)
   - P(x) = M(x) + k bits

How can we determine these k bits?

Properties of Polynomial Arithmetic

- Divisor
  - Any polynomial B(x) can be divided by a polynomial C(x) if B(x) is of the same or higher degree than C(x)
- Remainder
  - The remainder obtained when B(x) is divided by C(x) is obtained by subtracting C(x) from B(x)
- Subtraction
  - To subtract C(x) from B(x), simply perform an XOR on each pair of matching coefficients
- For example: $(x^3+1)/(x^3+x^2+1) =$?

CRC - Sender

- Given
  - M(x) = 10011010 = $x^7 + x^4 + x^3 + x$
  - C(x) = 1101 = $x^3 + x^2 + 1$
- Steps
  - $T(x) = M(x) * x^k$ (add zeros to increase degree of M(x) by k)
  - Find remainder, R(x), from T(x)/C(x)
  - $P(x) = T(x) – R(x) \Rightarrow$ M(x) followed by R(x)
- Example
  - T(x) = 10011010000
  - R(x) = 101
  - P(x) = 10011010101

CRC - Receiver

- Receive Polynomial P(x) + E(x)
  - E(x) represents errors
  - (if no errors then E(x) = 0)
- Divide (P(x) + E(x)) by C(x)
  - If result = 0, either
    - No errors (E(x) = 0, and P(x) is evenly divisible by C(x))
    - (P(x) + E(x)) is exactly divisible by C(x), error will not be detected

CRC – Example Encoding

C(x) =	$x^3 + x^2 + 1$	= 1101	Generator
M(x) =	$x^7 + x^4 + x^3 + x$	= 10011010	Message

```
1101 | 10011010000   <- Message plus k zeros
        1101
        ----
         1001
         1101
         ----
          1000
          1101
          ----
           1011
           1101
           ----
            1100
            1101
            ----
             1000
             1101
             ----
              101   <- Remainder m mod c
```

1101: k + 1 bit check sequence c, equivalent to a degree-k polynomial

Remainder

m mod c

Result:

Transmit message followed by remainder:

10011010101

CRC – Example Decoding – No Errors

C(x) =	$x^3 + x^2 + 1$	= 1101	Generator
P(x) =	$x^{10} + x^7 + x^6 + x^4 + x^2 + 1$	= 10011010101	Received Message

```
1101 | 10011010101
        1101
        ----
         1001
         1101
         ----
          1000
          1101
          ----
           1011
           1101
           ----
            1100
            1101
            ----
             1101
             1101
             ----
                0
```

k + 1 bit check sequence c, equivalent to a degree-k polynomial (→ 1101)

Received message, no errors (→ 10011010101)

Remainder m mod c (→ 0)

Result:

CRC test is passed

CRC – Example Decoding – with Errors

$C(x) = x^3 + x^2 + 1 = 1101$ Generator

$P(x) = x^{10} + x^7 + x^5 + x^4 + x^2 + 1 = 10010110101$ Received Message

```
1101 | 10010110101   <-- Received message
        1101          <-- Two bit errors
        ----
         1000
         1101
         ----
          1011
          1101
          ----
           1101
           1101
           ----
            0101
```

k + 1 bit check sequence c, equivalent to a degree-k polynomial

Remainder

m mod c

Result:

CRC test failed

CRC Error Detection

- Properties
  - Characterize error as E(x)
  - Error detected unless C(x) divides E(x)
    - (*i.e.*, E(x) is a multiple of C(x))

Example of Polynomial Multiplication

- Multiply
  - 1101 by 10110
  - $x^3 + x^2 + 1$ by $x^4 + x^2 + x$

```
  1011
 10110
 -----
  1101
 1101
1101
---------
00011111110
```

This is a multiple of c, so that if errors occur according to this sequence, the CRC test would be passed

On Polynomial Arithmetic

- Polynomial arithmetic
  - A fancy way to think about addition with no carries.
  - Helps in the determination of a good choice of $C(x)$
  - A non-zero vector is not detected if and only if the error polynomial $E(x)$ is a multiple of $C(x)$
- Implication
  - Suppose $C(x)$ has the property that $C(1) = 0$ (i.e. $(x + 1)$ is a factor of $C(x)$)
  - If $E(x)$ corresponds to an undetected error pattern, then it must be that $E(1) = 0$
  - Therefore, any error pattern with an odd number of error bits is detected

CRC Error Detection

- What errors can we detect?
  - All single-bit errors, if x^k and x^0 have non-zero coefficients
  - All double-bit errors, if C(x) has at least three terms
  - All odd bit errors, if C(x) contains the factor (x + 1)
  - Any bursts of length < k, if C(x) includes a constant term
  - Most bursts of length ≥ k

Common Polynomials for C(x)

CRC	C(x)
CRC-8	$x^8 + x^2 + x^1 + 1$
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^1 + 1$
CRC-12	$x^{12} + x^{11} + x^3 + x^2 + x^1 + 1$
CRC-16	$x^{16} + x^{15} + x^2 + 1$
CRC-CCITT	$x^{16} + x^{12} + x^5 + 1$
CRC-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x^1 + 1$

CRC Error Detection

- Proof of odd-bit detection
  - Assume C(x) has the form C′(x) (x + 1)
  - Implies P(x) = C(x) f(x) = (x + 1) C′(x) f(x)
  - P(1) = (0) C′(1) f(1) = 0
  - What if P(x) + E(x) received?
    - Detected if P(1) + E(1) = E(1) is not 0
    - E(1) is 0 iff E(x) has an even number of terms

CRC Error Detection

- Proof of burst errors of less than k bits
  - Assume that $C(x) = x^k + \ldots + 1$
  - Write $E(x) = x^i (x^{k-1} + \ldots + 1)$
  - No power of x can be factored out of C(x)
  - C(x) is not a factor of x^i
  - C(x) cannot be a factor of polynomial with smaller degree

Error Detection vs. Error Correction

- Detection
  - Pro: Overhead only on messages with errors
  - Con: Cost in bandwidth and latency for retransmissions
- Correction
  - Pro: Quick recovery
  - Con: Overhead on all messages
- What should we use?
  - Correction if retransmission is too expensive
  - Correction if probability of errors is high