CS425 /CSE424/ECE428 – Distributed Systems – Fall 2011

Consensus

Material derived from slides by I. Gupta, M. Harandi, J. Hou, S. Mitra, K. Nahrstedt, N. Vaidya

Give it a thought

- Have you ever wondered why software vendors always only offer solutions that promise five-9's reliability, seven-9's reliability, but never 100% reliability?
- The fault does not lie with Microsoft Corp. or Apple Inc. or Cisco
- The fault lies in the impossibility of consensus

Consensus: Example

- Proposal: move CS425 exam date up from Dec 16th
- Consensus needed
  - All students must be OK with new date (input)
  - Everyone must know the final decision (agreement)

What is Consensus?

- N processes
- Each process p has
  - input variable x_p : initially either 0 or 1
  - output variable y_p : initially b (b=undecided) – can be changed only once
- Consensus problem: design a protocol so that either
  - all non-faulty processes set their output variables to 0
  - Or non-faulty all processes set their output variables to 1
  - There is at least one initial state that leads to each outcomes 1 and 2 above

Solve Consensus!

- Uh, what's the model? (assumptions!)
- Processes fail only by crash-stopping
- Synchronous system: bounds on
  - Message delays
  - Max time for each process step
  - e.g., multiprocessor (common clock across processors)
- Asynchronous system: no such bounds!
- e.g., The Internet! The Web!

Consensus in Synchronous Systems

- For a system with at most f processes crashing, the algorithm proceeds in f+1 rounds (with timeout), using basic multicast (B-multicast).
- $Values^r_i$: the set of proposed values known to process p=P_i at the beginning of round r.
- Initially $Values^0_i = \{\}$; $Values^1_i = \{v_i=x_p\}$

```
for round r = 1 to f+1 do
    multicast (Values^r_i)
     Values^(r+1)_i ← Values^r_i
    for each V_j received
     Values^(r+1)_i = Values^(r+1)_i  ∪ V_j
    end
end
y_p=d_i = minimum(Values^(f+1)_i)
```

Why does the Algorithm Work?

- Proof by contradiction.
- Assume that two non-faulty processes differ in their final set of values.
- Suppose p_i and p_j are these processes.
- Assume that p_i possesses a value v that p_j does not possess.
  - → In the last round, some third process, p_k, sent v to p_i, and crashed before sending v to p_j.
  - → Any process sending v in the penultimate round must have crashed; otherwise, both p_k and p_j should have received v.
  - → Proceeding in this way, we infer at least one crash in each of the preceding rounds.
  - → But we have assumed at most f crashes can occur and there are f+1 rounds ==> contradiction.

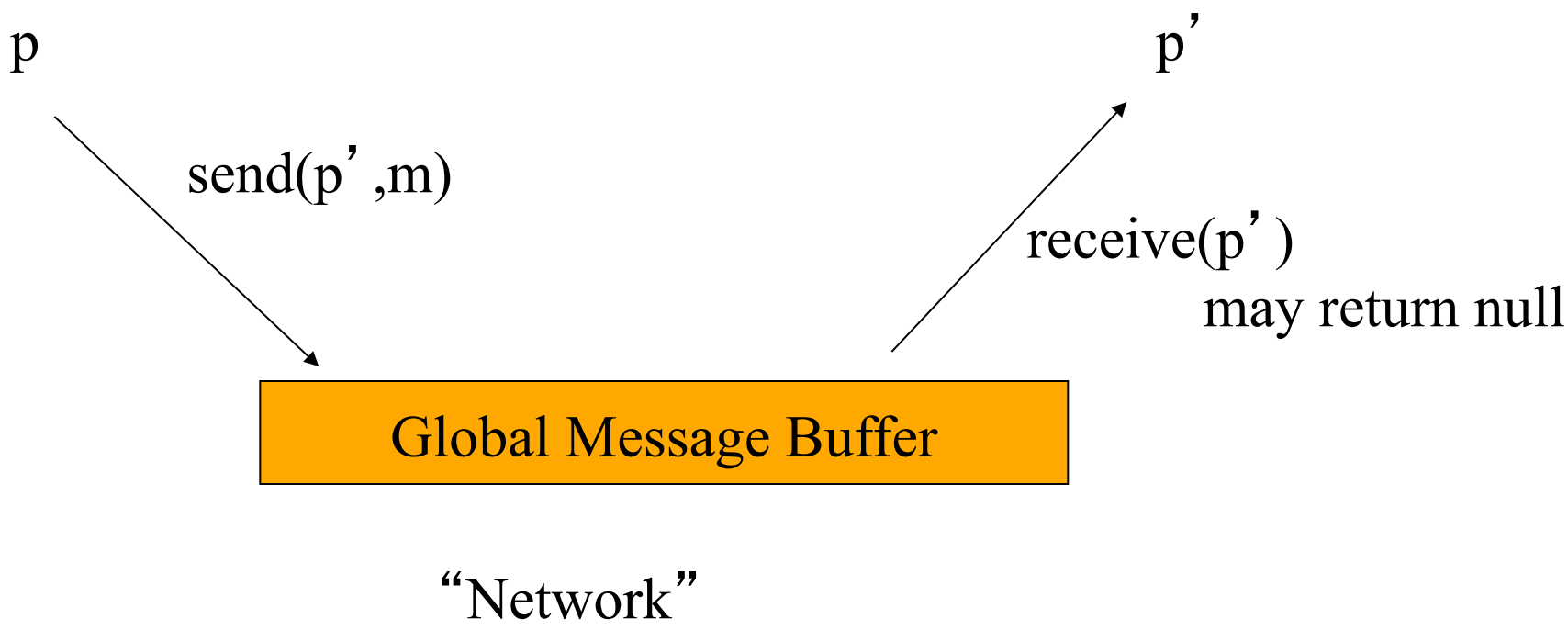
Consensus in an Asynchronous System

- Messages have arbitrary delay, processes arbitrarily slow
- Impossible to achieve!
  - even a single failed is enough to avoid the system from reaching agreement!
  - a slow process indistinguishable from a crashed process
- Impossibility Applies to any protocol that claims to solve consensus!

- Proved in a now-famous result by Fischer, Lynch and Patterson, 1983 (FLP)
  - Stopped many distributed system designers dead in their tracks
  - A lot of claims of "reliability" vanished overnight

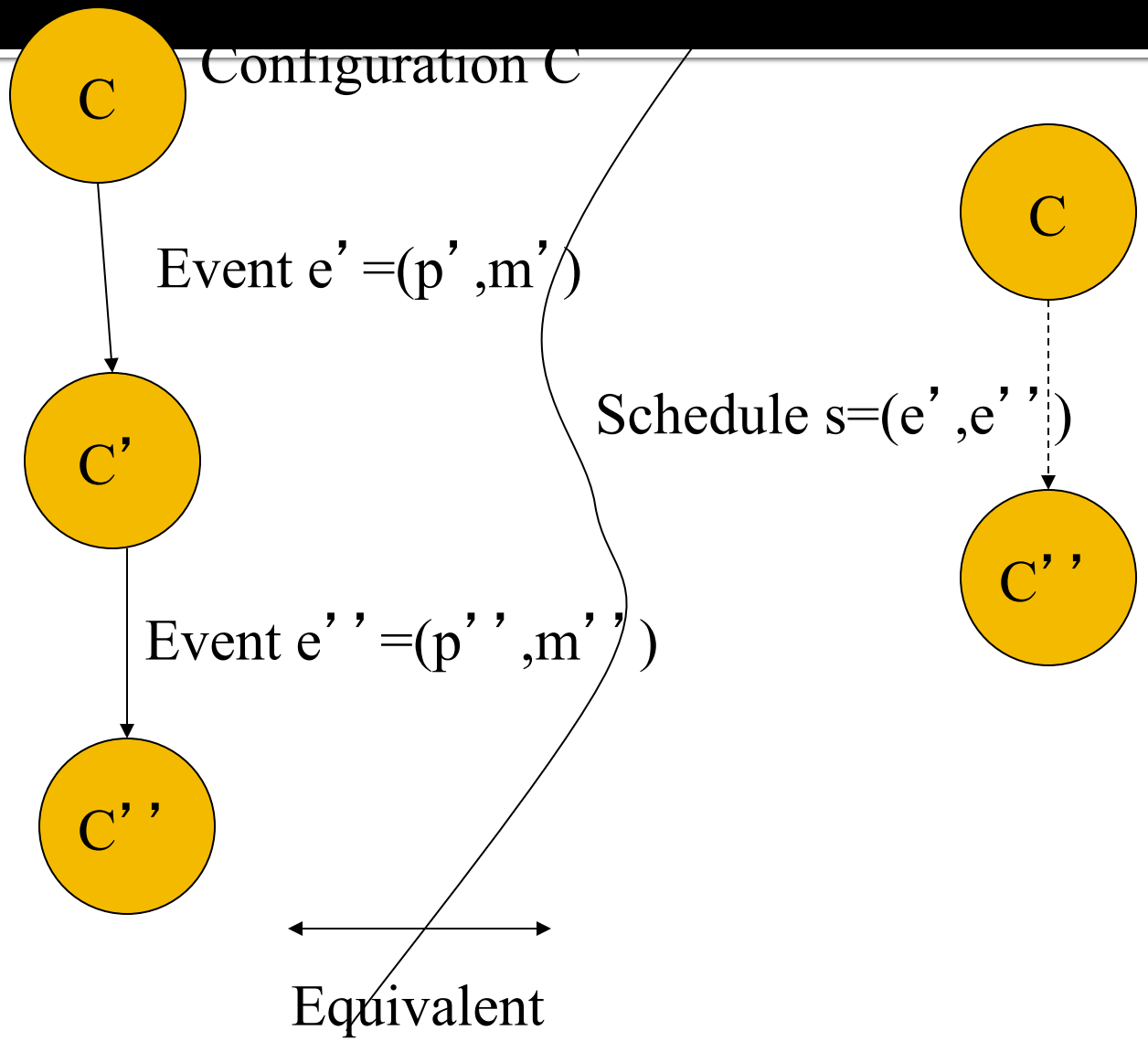
Recall

- Each process p has a state
  - program counter, registers, stack, local variables
  - input register xp : initially either 0 or 1
  - output register yp : initially b (b=undecided)
- Consensus Problem: design a protocol so that either
  - all non-faulty processes set their output variables to 0
  - Or non-faulty all processes set their output variables to 1
  - (No trivial solutions allowed)


p
p'
send(p',m)
receive(p')
may return null
Global Message Buffer
"Network"

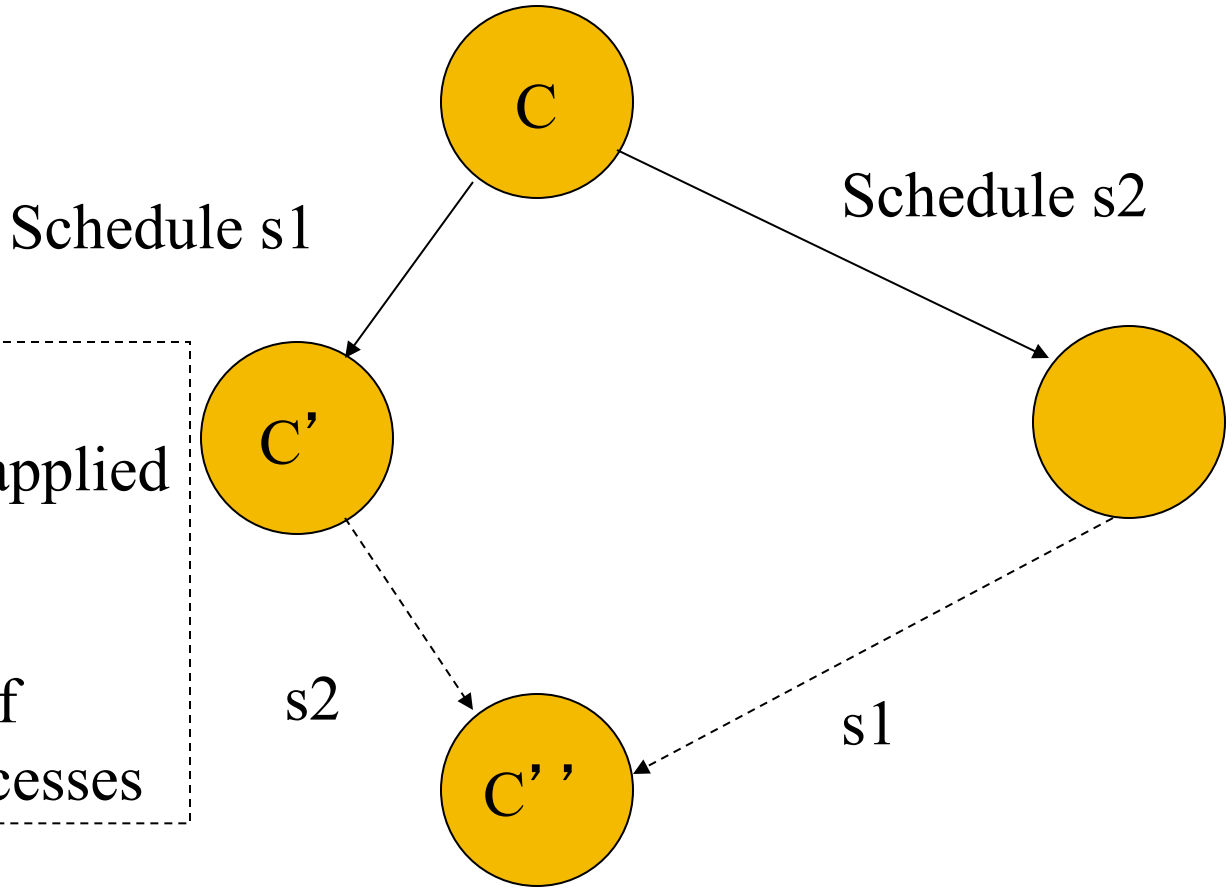
Different Definition of "State"

- State of a process
- Configuration: = Global state. Collection of states, one per process; and state of the global buffer
- Each Event consists atomically of three sub-steps:
  - receipt of a message by a process (say p), and
  - processing of message, and
  - sending out of all necessary messages by p (into the global message buffer)
- Note: this event is different from the Lamport events
- Schedule: sequence of events


C
Configuration C
Event e’ =(p’ ,m’)
C’
Event e’ ’ =(p’ ’ ,m’ ’)
C’ ’
C
Schedule s=(e’ ,e’ ’)
C’ ’
Equivalent

Lemma 1

Schedules are commutative



s1 and s2
- can each be applied to C
- involve <u>disjoint</u> sets of receiving processes

State Valencies

- Let config. C have a set of decision values V reachable from it
  - If |V| = 2, config. C is bivalent
  - If |V| = 1, config. C is said to be 0-valent or 1-valent, as is the case
- Bivalent means outcome is unpredictable

What we'll Show

- There exists an initial configuration that is bivalent
- Starting from a bivalent config., there is always another bivalent config. that is reachable

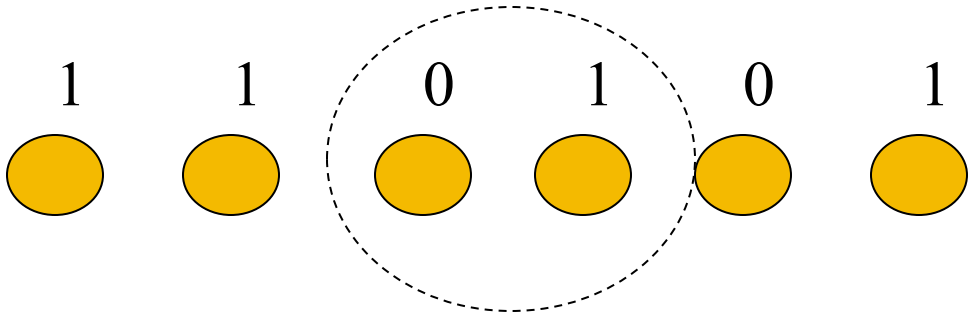
Lemma 2

- Some initial configuration is bivalent

•Suppose all initial configurations were either 0-valent or 1-valent.

•Place all configurations side-by-side, where adjacent configurations differ in initial xp value for *exactly one* process.

•Creates a lattice of states



•There *has* to be some adjacent pair of 1-valent and 0-valent configs.

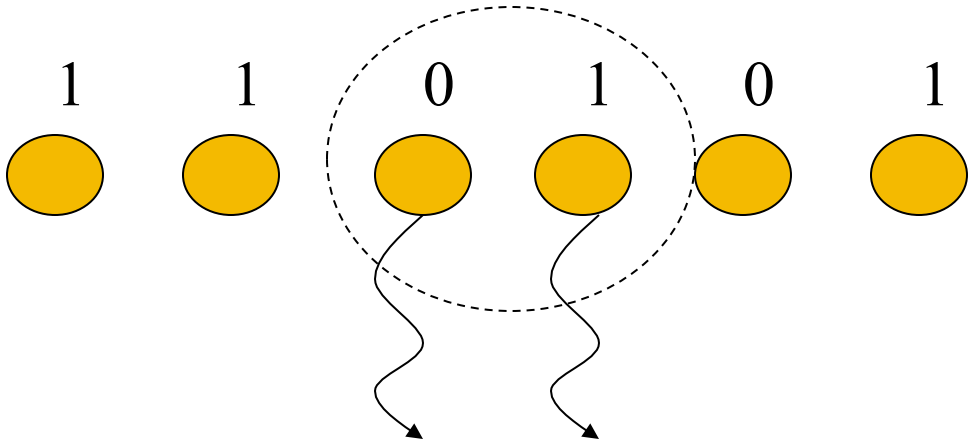
Lemma 2

- Some initial configuration is bivalent

•There has to be some adjacent pair of 1-valent and 0-valent configs.

•Let the process p be the one with a different state across these two configs.

•Now consider the world where process p has crashed



Both these initial configs. are *indistinguishable*. But one gives a 0 decision value. The other gives a 1 decision value.

So, both these initial configs. are bivalent when there is a failure

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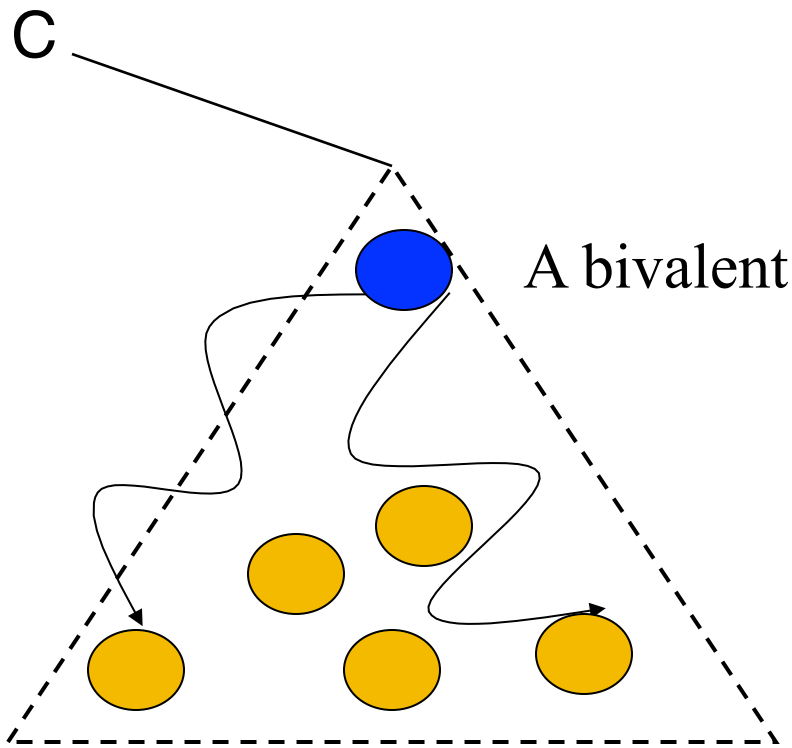
Lemma 3

- Starting from a bivalent config., there is always another bivalent config. that is reachable

Lemma 3

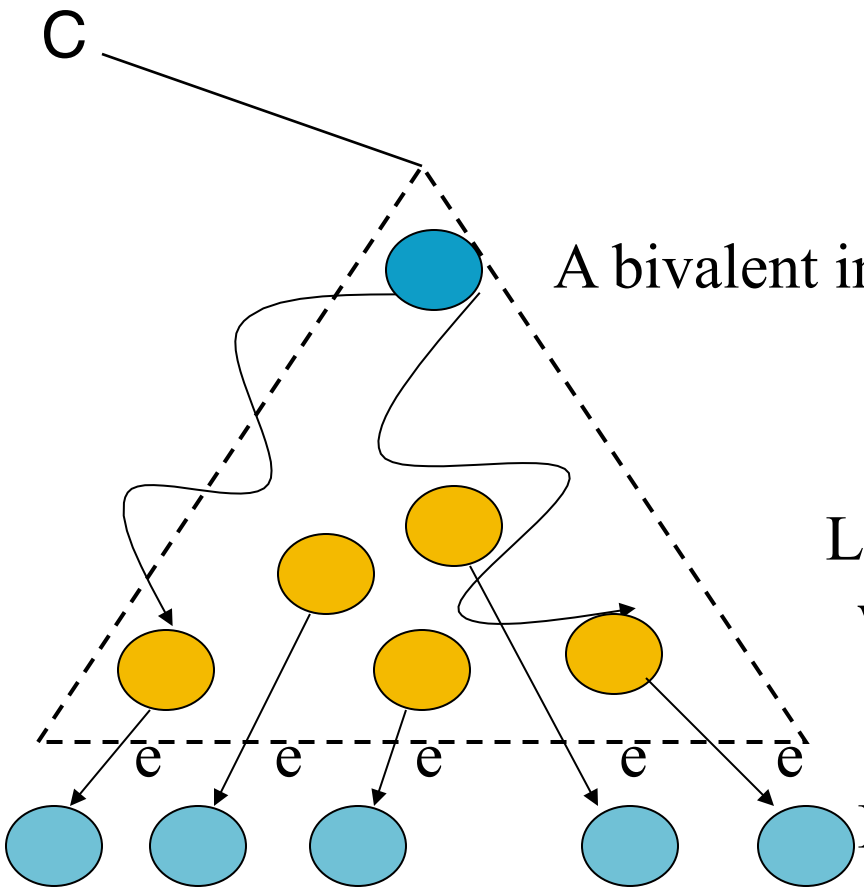


A bivalent initial config.

let e=(p,m) be an applicable event to the initial config.

Let C be the set of configs. reachable <u>without</u> applying e

Lemma 3



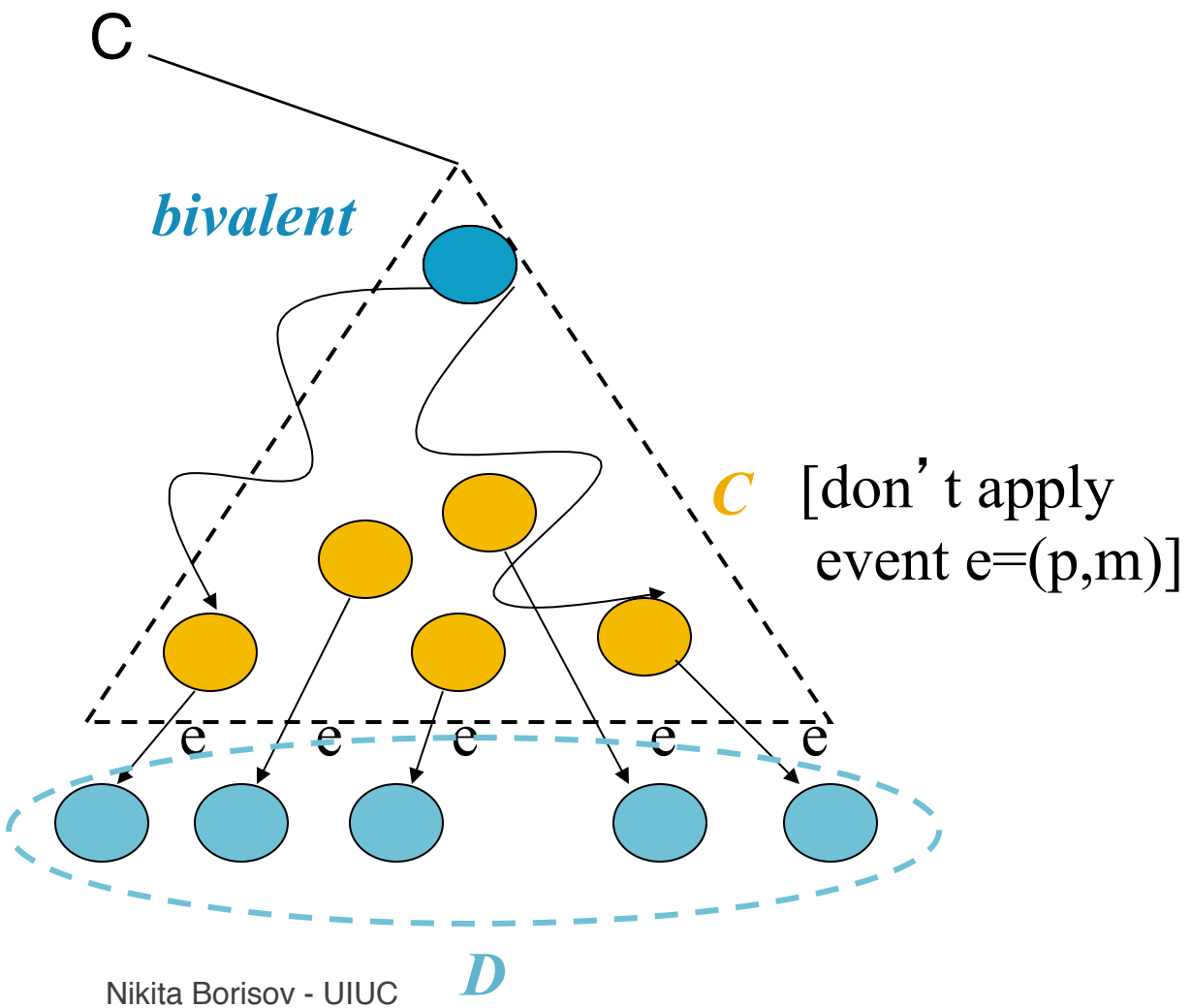
A bivalent initial config.

let e=(p,m) be an applicable event to the initial config.

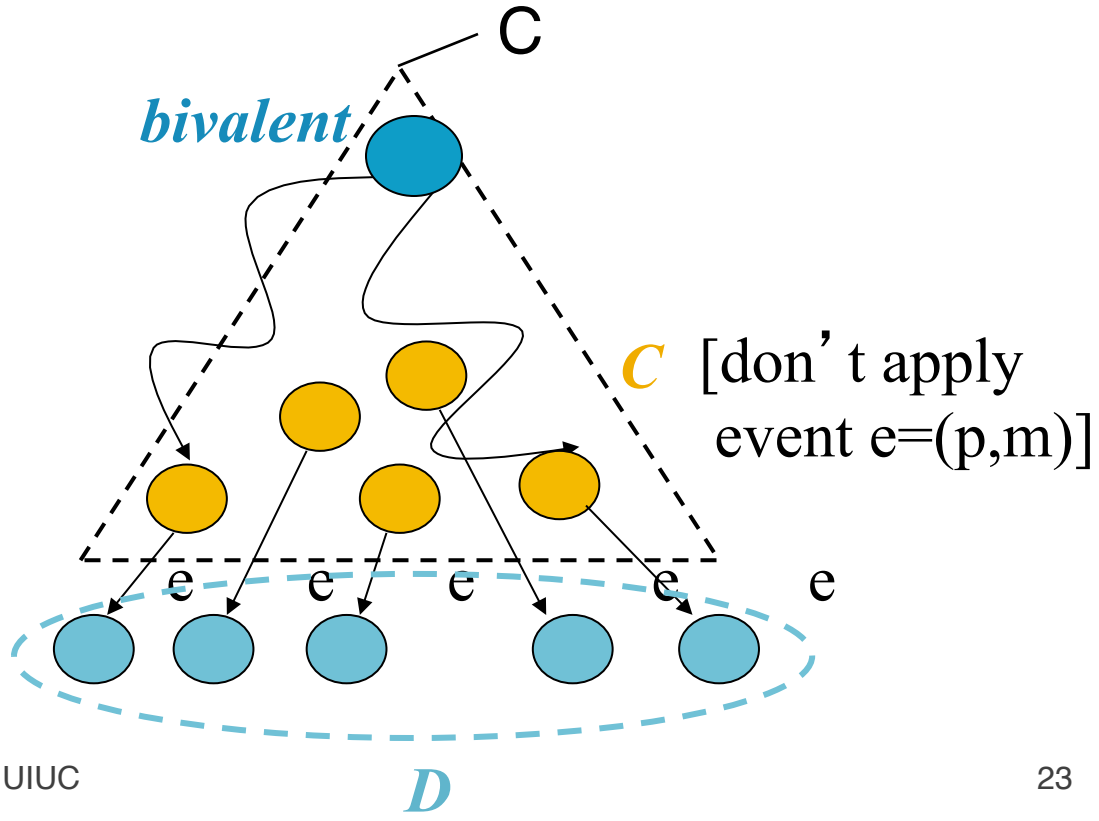
Let C be the set of configs. reachable without applying e

Let ***D*** be the set of configs. obtained by applying single event e to any config. in ***C***

Lemma 3

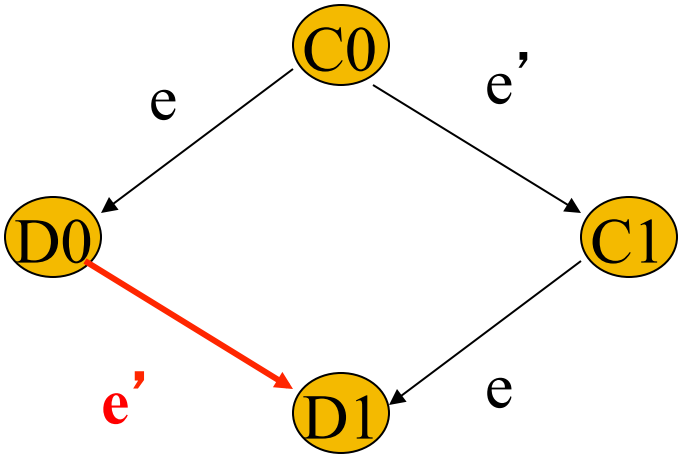


- Claim. Set D contains a bivalent config.
- Proof. By contradiction. That is, suppose D has only 0- and 1-valent states (and no bivalent ones)
- There are states D0 and D1 in D, and C0 and C1 in C such that
  - D0 is 0-valent, D1 is 1-valent
  - D0=C0 foll. by e=(p,m)
  - D1=C1 foll. by e=(p,m)
  - And C1 = C0 followed by some event e' =(p' ,m')
- (why?)

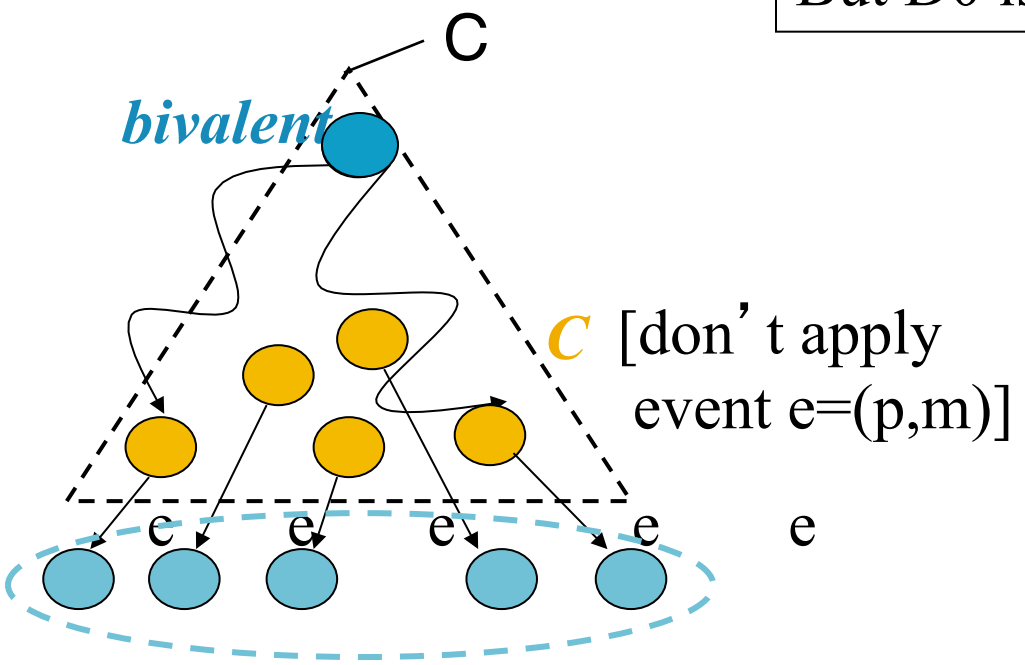


Proof. (contd.)

- Case I: p' is not p
- Case II: p' same as p



Why? (Lemma 1)
But D0 is then bivalent!



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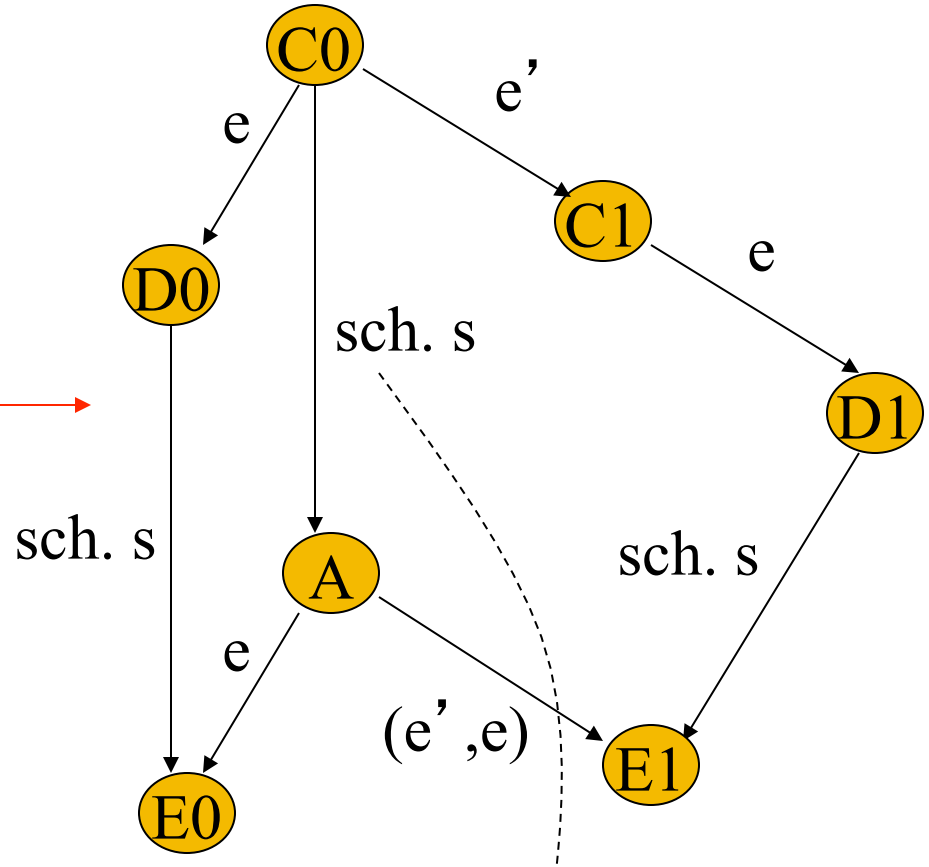

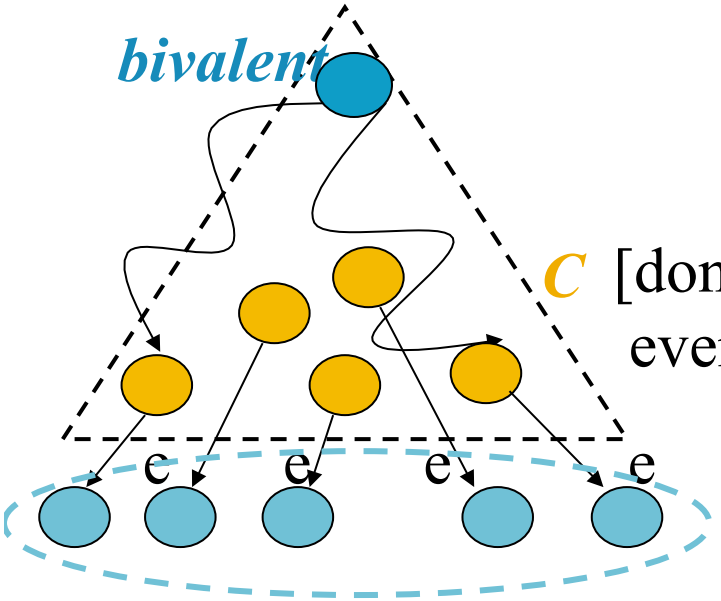

C [don't apply event e=(p,m)]

e

D

- finite
- deciding run from C0 (i.e., A is not bivalent)
- p takes no steps

But A is then bivalent!

Lemma 3

Starting from a bivalent config., there is always another bivalent config. that is reachable

Putting it all Together

- Lemma 2: There exists an initial configuration that is bivalent
- Lemma 3: Starting from a bivalent config., there is always another bivalent config. that is reachable

- Theorem (Impossibility of Consensus): There is always a run of events in an asynchronous distributed system (given any algorithm) such that the group of processes never reaches consensus (i.e., always stays bivalent)
  - "The devil's advocate always has a way out"

Why is Consensus Important? –

- Many problems in distributed systems are equivalent to (or harder than) consensus!
  - Agreement, e.g., on an integer (harder than consensus, since it can be used to solve consensus) is impossible!
  - Leader election is impossible!
    - A leader election algorithm can be designed using a given consensus algorithm as a black box
    - A consensus protocol can be designed using a given leader election algorithm as a black box
  - Accurate Failure Detection is impossible!
    - Should I mark a process that has not responded for the last 60 seconds as failed? (It might just be very, very, slow)
    - Completeness + Accuracy impossible to guarantee

Summary

- Consensus Problem
  - agreement in distributed systems
  - Solution exists in synchronous system model (e.g., supercomputer)
  - Impossible to solve in an asynchronous system (e.g., Internet, Web)
    - Key idea: with only one process failure and arbitrarily slow processes, there are always sequences of events for the system to decide any which way. Regardless of which consensus algorithm is running underneath.
  - FLP impossibility proof