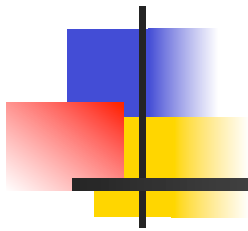


Programming Languages and Compilers (CS 421)



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Based in part on slides by Mattox Beckman, as updated by Vikram Adve, Gul Agha, and Elsa Gunter



Interpreter

- Takes abstract syntax trees as input
 - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
 - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next “state”
 - To get final value, put in a loop



Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like
$$(C, m) \dashrightarrow (C', m') \quad \text{or} \quad (C, m) \dashrightarrow m'$$
- C, C' is code remaining to be executed
- m, m' represent the state/store/memory/environment
 - Partial mapping from identifiers to values
 - Sometimes m (or C) not needed
- Indicates exactly one step of computation



Expressions and Values

- C, C' used for commands; E, E' for expressions; U, V for values
- Special class of expressions designated as *values*
 - Eg 2, 3 are values, but $2+3$ is only an expression
- Memory only holds values
 - Other possibilities exist

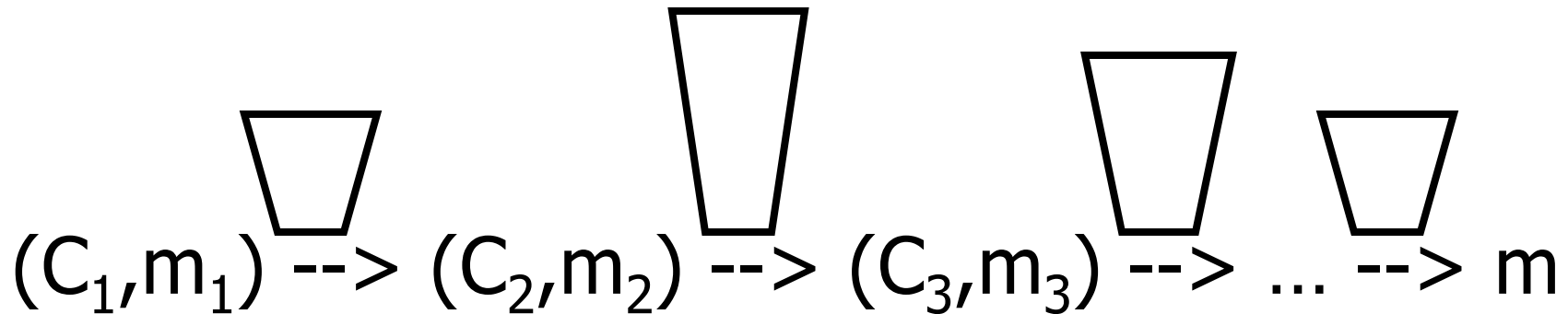


Evaluation Semantics

- Transitions successfully stops when E/C is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

Transition Semantics Evaluation

- A sequence of steps with trees of justification for each step



- Let \dashrightarrow^* be the transitive closure of \dashrightarrow
- Ie, the smallest transitive relation containing \dashrightarrow



Simple Imperative Programming Language

- $I \in \text{Identifiers}$
- $N \in \text{Numerals}$
- $B ::= \text{true} \mid \text{false} \mid B \ \& \ B \mid B \ \text{or} \ B \mid \text{not} \ B \mid E < E \mid E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- $C ::= \text{skip} \mid C; C \mid I ::= E \mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od}$



Transitions for Expressions

- Numerals are values
- Boolean values = {true, false}
- Identifiers: $(I, m) \dashrightarrow (m(I), m)$



Boolean Operations:

- Operators: (short-circuit)

$$\begin{array}{l}
 (\text{false} \ \& \ B, \ m) \ \rightarrow (\text{false}, m) \\
 (\text{true} \ \& \ B, \ m) \ \rightarrow (B, m) \\
 \\
 (\text{true} \ \text{or} \ B, \ m) \ \rightarrow (\text{true}, m) \\
 (\text{false} \ \text{or} \ B, \ m) \ \rightarrow (B, m) \\
 \\
 (\text{not true}, m) \ \rightarrow (\text{false}, m) \\
 (\text{not false}, m) \ \rightarrow (\text{true}, m)
 \end{array}
 \quad
 \begin{array}{l}
 \frac{(B, \ m) \ \rightarrow (B'', \ m)}{(B \ \& \ B', \ m) \ \rightarrow (B'' \ \& \ B', \ m)} \\
 \\
 \frac{(B, \ m) \ \rightarrow (B'', \ m)}{(B \ \text{or} \ B', \ m) \ \rightarrow (B'' \ \text{or} \ B', \ m)} \\
 \\
 \frac{(B, \ m) \ \rightarrow (B', \ m)}{(\text{not } B, \ m) \ \rightarrow (\text{not } B', \ m)}
 \end{array}$$



Relations

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \sim E', m) \dashrightarrow (E'' \sim E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \sim E, m) \dashrightarrow (V \sim E', m)}$$

$(U \sim V, m) \dashrightarrow (\text{true}, m)$ or (false, m)
depending on whether $U \sim V$ holds or not



Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \text{ op } E', m) \dashrightarrow (E'' \text{ op } E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \text{ op } E, m) \dashrightarrow (V \text{ op } E', m)}$$

$(U \text{ op } V, m) \dashrightarrow (N, m)$ where N is the specified value for $U \text{ op } V$



Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory



Commands

$$(\text{skip}, m) \dashrightarrow m$$
$$(E, m) \dashrightarrow (E', m)$$
$$\frac{(E, m) \dashrightarrow (E', m)}{(I ::= E, m) \dashrightarrow (I ::= E', m)}$$
$$(I ::= V, m) \dashrightarrow m[I \leftarrow V]$$
$$\frac{(C, m) \dashrightarrow (C'', m')}{(C; C', m) \dashrightarrow (C''; C', m')}$$
$$\frac{(C, m) \dashrightarrow m'}{(C; C', m) \dashrightarrow (C', m')}$$
$$(C; C', m) \dashrightarrow (C''; C', m')$$



If Then Else Command - in English

- If the boolean guard in an `if_then_else` is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.



If Then Else Command

$(\text{if true then } C \text{ else } C' \text{ fi, } m) \dashrightarrow (C, m)$

$(\text{if false then } C \text{ else } C' \text{ fi, } m) \dashrightarrow (C', m)$

$$\frac{(B, m) \dashrightarrow (B', m)}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi, } m) \dashrightarrow (\text{if } B' \text{ then } C \text{ else } C' \text{ fi, } m)}$$



While Command

(while B do C od, m)

--> (if B then C ; while B do C od else skip fi,
 m)

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.



Example Evaluation

- First step:

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)
 $\rightarrow ?$



Example Evaluation

- First step:

$$(x > 5, \{x \rightarrow 7\}) \dashrightarrow ?$$

$$\begin{aligned} &(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ &\quad \{x \rightarrow 7\}) \\ &\quad \dashrightarrow ? \end{aligned}$$



Example Evaluation

- First step:

$$(x, \{x \rightarrow 7\}) \dashrightarrow (7, \{x \rightarrow 7\})$$

$$(x > 5, \{x \rightarrow 7\}) \dashrightarrow ?$$

$$\begin{aligned} &(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ &\quad \{x \rightarrow 7\}) \\ &\quad \dashrightarrow ? \end{aligned}$$



Example Evaluation

- First step:

$$(x, \{x \rightarrow 7\}) \rightarrow (7, \{x \rightarrow 7\})$$

$$\frac{(x > 5, \{x \rightarrow 7\}) \rightarrow (7 > 5, \{x \rightarrow 7\})}{(if\ x > 5\ then\ y := 2 + 3\ else\ y := 3 + 4\ fi,$$

$$\{x \rightarrow 7\})$$

$$\rightarrow ?$$



Example Evaluation

- First step:

$$(x, \{x \rightarrow 7\}) \dashrightarrow (7, \{x \rightarrow 7\})$$

$$\frac{(x > 5, \{x \rightarrow 7\}) \dashrightarrow (7 > 5, \{x \rightarrow 7\})}{(if\ x > 5\ then\ y := 2 + 3\ else\ y := 3 + 4\ fi,$$

$$\{x \rightarrow 7\})$$

$$\dashrightarrow (if\ 7 > 5\ then\ y := 2 + 3\ else\ y := 3 + 4\ fi,$$
$$\{x \rightarrow 7\})$$



Example Evaluation

- Second Step:

$$\frac{(7 > 5, \{x \rightarrow 7\}) \rightarrow (\text{true}, \{x \rightarrow 7\})}{(\text{if } 7 > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\})}$$
$$\rightarrow (\text{if true then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\})$$

- Third Step:

$$(\text{if true then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\})$$
$$\rightarrow (y := 2 + 3, \{x \rightarrow 7\})$$



Example Evaluation

- Fourth Step:

$$\frac{(2+3, \{x \rightarrow 7\}) \rightarrow (5, \{x \rightarrow 7\})}{(y := 2+3, \{x \rightarrow 7\}) \rightarrow (y := 5, \{x \rightarrow 7\})}$$

- Fifth Step:

$$(y := 5, \{x \rightarrow 7\}) \rightarrow \{y \rightarrow 5, x \rightarrow 7\}$$



Example Evaluation

- Bottom Line:

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)

--> (if $7 > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)

--> (if true then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)

--> ($y := 2 + 3$, $\{x \rightarrow 7\}$)

--> ($y := 5$, $\{x \rightarrow 7\}$) --> $\{y \rightarrow 5, x \rightarrow 7\}$



Adding Local Declarations

- Add to expressions:
- $E ::= \dots \mid \text{let } I = E \text{ in } E' \mid \text{fun } I \rightarrow E \mid E E'$
- $\text{fun } I \rightarrow E$ is a value
- Could handle local binding using state, but have assumption that evaluating expressions doesn't alter the environment
- We will use substitution here instead
- **Notation:** $E[E' / I]$ means replace all free occurrence of I by E' in E



Call-by-value (Eager Evaluation)

$$(\text{let } I = V \text{ in } E, m) \dashrightarrow (E[V/I], m)$$

$$(E, m) \dashrightarrow (E'', m)$$

$$\frac{(E, m) \dashrightarrow (E'', m)}{(\text{let } I = E \text{ in } E', m) \dashrightarrow (\text{let } I = E'' \text{ in } E')}$$

$$((\text{fun } I \rightarrow E) V, m) \dashrightarrow (E[V/I], m)$$

$$(E', m) \dashrightarrow (E'', m)$$

$$\frac{(E', m) \dashrightarrow (E'', m)}{((\text{fun } I \rightarrow E) E', m) \dashrightarrow ((\text{fun } I \rightarrow E) E'', m)}$$

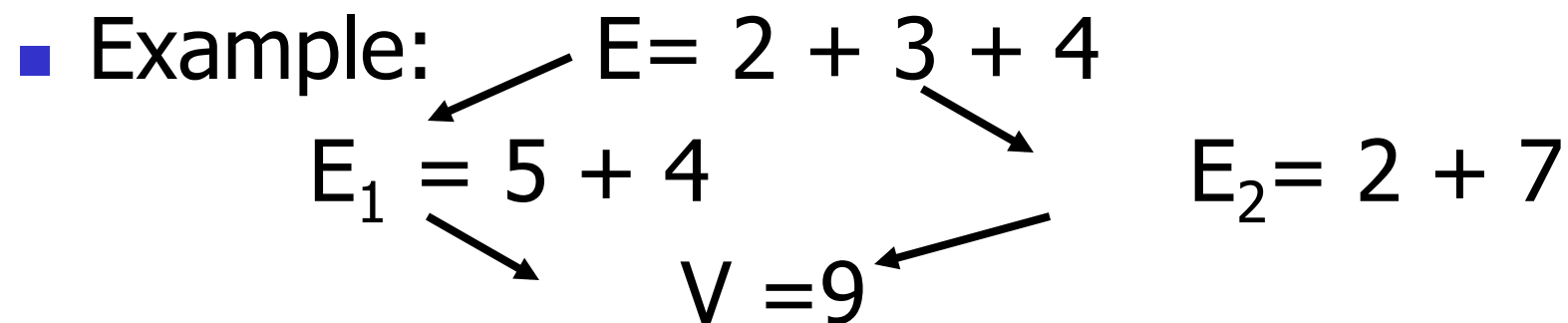


Call-by-name (Lazy Evaluation)

- $(\text{let } I = E \text{ in } E', m) \dashrightarrow (E' [E / I], m)$
- $((\text{fun } I \rightarrow E') E, m) \dashrightarrow (E' [E / I], m)$
- Question: Does it make a difference?
- It can depending on the language

Church-Rosser Property

- Church-Rosser Property: If $E \rightarrow^* E_1$ and $E \rightarrow^* E_2$, if there exists a value V such that $E_1 \rightarrow^* V$, then $E_2 \rightarrow^* V$
- Also called **confluence** or **diamond property**





Does It always Hold?

- No. Languages with side-effects tend not be Church-Rosser with the combination of call-by-name and call-by-value
- Alonzo Church and Barkley Rosser proved in 1936 the λ -calculus does have it
- Benefit of Church-Rosser: can check equality of terms by evaluating them (Given evaluation strategy might not terminate, though)