Lecture 3 — User-defined types

- **In this lecture, you will learn to define trees in OCaml (analogous to what you might do in Java or C++ by defining a `tree` class). This will allow us to define *abstract syntax trees*, which we will use extensively in this class. Abstract syntax trees are the central data structure in a compiler.**

- **Specific topics:**
  - **User-defined types in OCaml**
  - **Defining trees (including ASTs)**

Type abbreviations in OCaml

- **OCaml allows new names to be introduced as abbreviations for types:**

  `type` t `=` te

- te **is a type expression:**

$$te = \texttt{int} \mid \texttt{float} \mid \ldots \mid te \; \texttt{*} \; te \; \texttt{*} \; \cdots \; \texttt{*} \; te$$
$$\mid te \; \texttt{list} \mid te \rightarrow te$$

- **Examples of type expressions:**

Defining new types

- **OCaml allows you to create new types by writing:**

  $$\texttt{type}\ t\ \texttt{=}\ C_1\ \texttt{[of}\ te_1\texttt{]}\ |\ \ldots\ |\ C_n\ \texttt{[of}\ te_n\texttt{]}$$

  where C_1, ..., C_n are *constructors* (identifiers starting with capital letters).

- **The above declaration creates a new type, called t, and *automatically* creates new functions that construct values of type t:**
  - $C_1 : te_1 \to t$
  - $\ldots$
  - $C_n : te_n \to t$

Defining new types (cont.)

- **For example, suppose we define this type:**

```
type form_of_id = License of string
                | SScard of int * int * int
                | Student_id of string
```

- **As soon as this is entered, you can enter:**

```
# let myid = Student_id "123456789";;
val myid : form_of_id = Student_id "123456789"
# let hisid = SScard (123, 45, 6789);;
val hisid : form_of_id = SScard (123, 45, 6789)
```

Defining new types (cont.)

- **Think of values of type t as tuples combined with a *tag* — a number between 1 and n — saying which kind of t-typed value it is.**

- **Functions on values of type t can be defined using pattern-matching:**

$$
\begin{array}{l}
\texttt{let f x = match x with} \\
\qquad\qquad\quad C_1(x,\ldots,y) \texttt{ -> } e_1 \\
\qquad\qquad\ \mid C_2(x,\ldots,y) \texttt{ -> } e_2 \\
\qquad\qquad\ \mid \ldots \\
\qquad\qquad\ \mid C_n(x,\ldots,y) \texttt{ -> } e_n
\end{array}
$$

Type definition example

```
type form_of_id = License of string
                | SScard of int * int * int
                | Student_id of string


let string_of_id id =
   match id with
       License s -> "license " ^ s
     | SScard (x,y,z) -> "ssnum " ^ (string_of_int x)
                    ^ (string_of_int y) ^ (string_of_int z)
     | Student_id s -> "uin " ^ s
```

Type definition exercise

- **Given type**

```
type shape = Circle of float
           | Square of float
           | Triangle of float * float * float
```

define function `string_of_shape`**:** `shape` $\rightarrow$ `string` **that prints the shape (e.g. outputs "**`circle 4.3`**" for a circle):**

```
let string_of_shape sh =
    match sh with
      Circle r ->

   |


   |
```

Recursive type definitions

- **In this type definition:**

$$\texttt{type}\ t\ \texttt{=}\ C_1\ \texttt{[of}\ te_1\texttt{]}\ \mid \ldots \mid\ C_n\ \texttt{[of}\ te_n\texttt{]}$$

the type expressions te_i can contain t, making the type declaration recursive. This allows for the definition of types like lists and trees, e.g.

```
type mylist = Empty | Cons of int * mylist
let list1 = Cons (3, Cons (4, Empty))
```

- **Ex: write the function** `sum` : `mylist` $\rightarrow$ `int`**.**

Defining trees

- **Binary trees (with integer labels):**

```
type bintree = Empty
             | Node of int * bintree * bintree

let tree1 = Node (3,
                  Node (6, Empty, Empty),
                  Node (7, Empty, Empty));;
```

- **Arbitrary trees (with integer labels):**

```
type tree = Node of int * tree list

let smalltree = Node (3, [])
let bigtree = Node (3, [Node(...), Node(...), ...])
```

Exercises: Functions on binary trees

```
type bintree = Empty
             | Node of int * bintree * bintree
```

- **Define isLeaf: bintree → bool**

```
let isLeaf t = match t with
    Empty ->

    Node(i, t1, t2) ->
```

- **Define sum: bintree → int**

```
let sum t = match t with
    Empty ->

    Node(i, t1, t2) ->
```

Polymorphic types

- **We can define a type of binary trees with labels of any type (but all the same type for any particular tree):**

```
type 'a bintree = Empty
                | Node of 'a * 'a bintree * 'a bintree

let x = Node("ben", Empty, Empty)
let y = Node(4.5, Empty, Empty)
```

- **The `sum` function defined above still works, when applied to a value of type `int bintree`.**

- `bintree`**s are homogeneous, e.g.**

```
Node("ben", Node(4, Empty, Empty), Empty)
```

gives a type error.

Mutually recursive types

- **Sometimes two user-defined types are mutually interdependent: values of either type can contain values of the other type. To define mutually-recursive types, give both type declarations separated by the word** `and`**:**

```
type ocamlexpr = Name of string | Intconst of int
               | Let of definition * ocamlexpr
and definition = Def of string * ocamlexpr
```

- **The above defines two types and four constructors:**
  - `Name: string` → `ocamlexpr`
  - `Intconst: int` → `ocamlexpr`
  - `Let: definition * ocamlexpr` → `ocamlexpr`
  - `Def: string * ocamlexpr` → `definition`

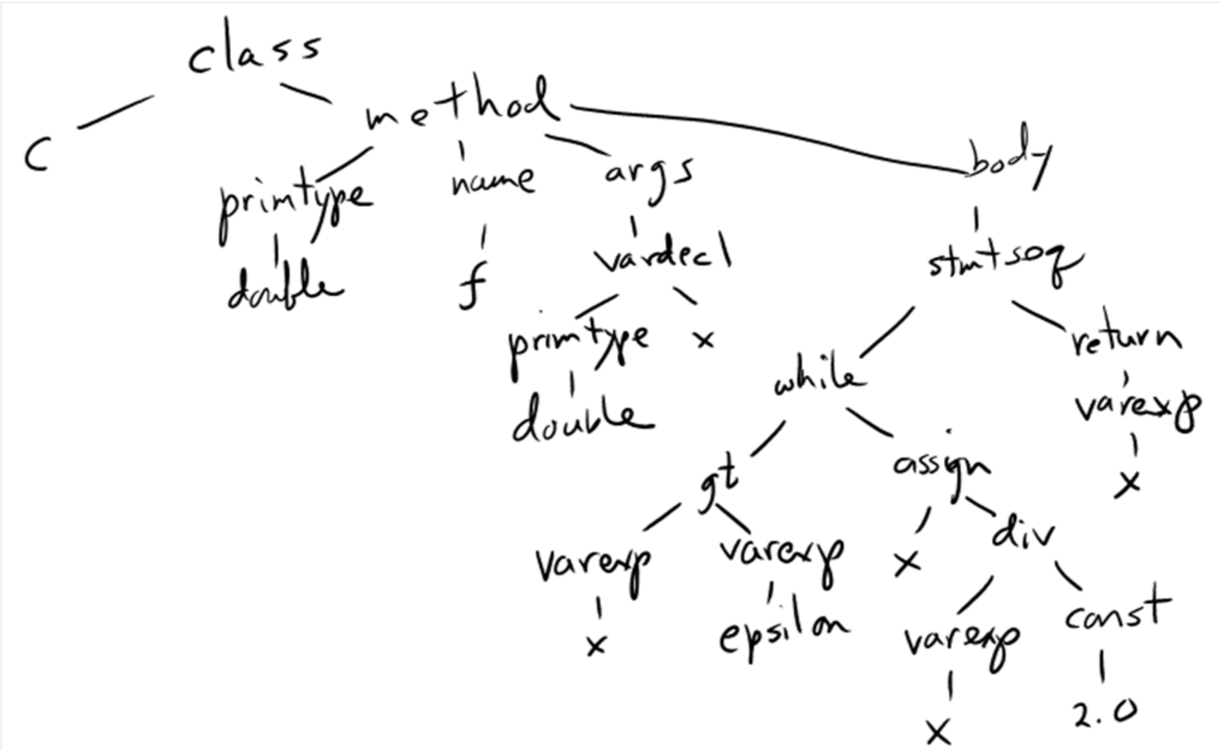
Abstract syntax

- *Abstract syntax* is a tree representation of the syntactic structure of programs.
- The internal nodes of abstract syntax trees are labelled with names, called *abstract syntax operators*; the leaf nodes are labelled with strings, ints, etc.
- The specific trees used to represent programs in a given language are determined by the person writing the language processor (e.g. compiler).
- For example, this program in Java:

```
class C {
   double f (double x) {
      while (x > epsilon) x = x/2.0;
      return x; } }
```

Abstract syntax (cont.)

would be represented by a tree something like this:



- **In OCaml, type definitions can be used to define abstract syntax, and pattern-matching can be used to define functions on abstract syntax trees.**

Ex: Abstract syntax of simple expressions

- **Here is an abstract syntax for simple arithmetic expressions as an OCaml data type:**

```
type expr = Int of int | Plus of expr*expr
          | Times of expr*expr | Negate of expr
```

- **For example:**
  - `Plus(Int 3, Int 5)` **is abstract syntax for** 3+5
  - `Plus(Int 3, Times(Int 5, Int 6))` **is abstract syntax for** 3+5*6 **or** 3+(5*6)
  - `Times(Plus(Int 3, Int 5), Int 6)` **is abstract syntax for** (3+5)*6

Exercises using `expr`

```
type expr = Int of int | Plus of expr*expr
          | Times of expr*expr | Negate of expr
```

- **Show the abstract syntax tree for expression `4+-(7*-8+4)`:**

- **Give the OCaml expression of type `expr` for that tree:**

Exercises using `expr` (cont.)

- **Write the function `countPluses: expr` $\rightarrow$ `int`, which counts the number of `Plus` operations in an `expr`:**

```
  type expr = Int of int | Plus of expr*expr
            | Times of expr*expr | Negate of expr
```

```
let rec countPluses e = match e with
  Int i ->

| Plus(e1, e2) ->

| Times(e1, e2) ->

| Negate e ->
```

Exercises using `expr` (cont.)

- **Write the function** `eval: expr` $\rightarrow$ `int`**, which evaluates its argument, e.g.** `eval (Times(Negate(Int 5), Int 6))` **= -30.**

```
let rec eval e = match e with
  Int i ->

| Plus(e1, e2) ->

| Times(e1, e2) ->

| Negate e ->
```

Wrap-up

- **Today we discussed:**
  - **How to define new types in OCaml**
  - **Especially trees**
  - **Especially abstract syntax trees**
- **We discussed it because:**
  - **ASTs are central to writing compilers**
- **In the next class, we will:**
  - **Do more programming with ASTs**
- **What to do now:**
  - **Just come back on Thursday...**