

Programming Languages and Compilers (CS 421)



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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



Natural Semantics

- Aka Structural Operational Semantics, aka “Big Step Semantics”
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$



Simple Imperative Programming Language

- $I \in \text{Identifiers}$
- $N \in \text{Numerals}$
- $B ::= \text{true} \mid \text{false} \mid B \ \& \ B \mid B \ \text{or} \ B \mid \text{not} \ B$
 $\mid E < E \mid E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- $C ::= \text{skip} \mid C; C \mid I ::= E$
 $\mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od}$



Natural Semantics of Atomic Expressions

- Identifiers: $(I, m) \Downarrow m(I)$
- Numerals are values: $(N, m) \Downarrow N$
- Booleans: $(\text{true}, m) \Downarrow \text{true}$
 $(\text{false}, m) \Downarrow \text{false}$



Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \ \& \ B', m) \Downarrow \text{false}}$$

$$\frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \ \& \ B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \ \text{or} \ B', m) \Downarrow \text{true}}$$

$$\frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \ \text{or} \ B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}}$$

$$\frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$



Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation \sim hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V



Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \text{ op } V = N}{(E \text{ op } E', m) \Downarrow N}$$

where N is the specified value for $U \text{ op } V$



Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment:
$$\frac{(E, m) \Downarrow V}{(I ::= E, m) \Downarrow m[I \leftarrow V]}$$

Sequencing:
$$\frac{(C, m) \Downarrow m' \quad (C', m') \Downarrow m''}{(C; C', m) \Downarrow m''}$$



If Then Else Command

$$\frac{(B, m) \Downarrow \text{true} \quad (C, m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B, m) \Downarrow \text{false} \quad (C', m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$



While Command

$$\frac{(B, m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B, m) \Downarrow \text{true} \quad (C, m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m') \Downarrow m''}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m''}$$



Example: If Then Else Rule

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}) \Downarrow ?$



Example: If Then Else Rule

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$



Example: Arith Relation

? > ? = ?

$(x, \{x \rightarrow 7\}) \Downarrow ? \quad (5, \{x \rightarrow 7\}) \Downarrow ?$

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}) \Downarrow ?$



Example: Identifier(s)

$7 > 5 = \text{true}$

$(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5$

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$



Example: Arith Relation

$7 > 5 = \text{true}$

$(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5$

$(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$



Example: If Then Else Rule

$7 > 5 = \text{true}$

$(x, \{x \rightarrow 7\}) \Downarrow 7$ $(5, \{x \rightarrow 7\}) \Downarrow 5$

$(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}$

$(y := 2 + 3, \{x \rightarrow 7\})$

$\Downarrow ?$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$



Example: Assignment

$$\begin{array}{c}
 7 > 5 = \text{true} \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow ?
 \end{array}
 \qquad
 \begin{array}{c}
 \hline
 (2+3, \{x \rightarrow 7\}) \Downarrow ? \\
 \hline
 (y := 2 + 3, \{x \rightarrow 7\}) \\
 \Downarrow ? \\
 \hline
 \end{array}$$



Example: Arith Op

? + ? = ?

(2, {x->7}) ↓? (3, {x->7}) ↓?

7 > 5 = true

(2+3, {x->7}) ↓?

(x, {x->7}) ↓7 (5, {x->7}) ↓5

(y := 2 + 3, {x->7})

(x > 5, {x->7}) ↓true

↓?

(if x > 5 then y := 2 + 3 else y := 3 + 4 fi,

{x->7}) ↓ ?



Example: Numerals

$$2 + 3 = 5$$

$$\frac{(2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3}{\quad}$$

$$7 > 5 = \text{true}$$

$$\frac{(2+3, \{x \rightarrow 7\}) \Downarrow ?}{\quad}$$

$$\frac{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{\quad}$$

$$(y := 2 + 3, \{x \rightarrow 7\})$$

$$(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}$$

$$\Downarrow ?$$

$$\frac{(x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}{\quad}$$



Example: Arith Op

$$2 + 3 = 5$$

$$\frac{(2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3}{\quad}$$

$$7 > 5 = \text{true}$$

$$\frac{(2+3, \{x \rightarrow 7\}) \Downarrow 5}{\quad}$$

$$\frac{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{\quad}$$

$$(y := 2 + 3, \{x \rightarrow 7\})$$

$$\frac{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}}{\quad}$$

$$\Downarrow ?$$

$$\frac{\quad}{\text{(if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}$$



Example: Assignment

$$2 + 3 = 5$$

$$\frac{(2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3}{\quad}$$

$$7 > 5 = \text{true}$$

$$\frac{(2+3, \{x \rightarrow 7\}) \Downarrow 5}{\quad}$$

$$\frac{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{\quad}$$

$$(y := 2 + 3, \{x \rightarrow 7\})$$

$$(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}$$

$$\Downarrow \{x \rightarrow 7, y \rightarrow 5\}$$

$$\frac{(x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}{\quad}$$



Example: If Then Else Rule

$$\begin{array}{c}
 2 + 3 = 5 \\
 \hline
 (2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3 \\
 \hline
 (2+3, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (y := 2 + 3, \{x \rightarrow 7\}) \\
 \Downarrow \{x \rightarrow 7, y \rightarrow 5\} \\
 \hline
 \begin{array}{c}
 7 > 5 = \text{true} \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\
 \hline
 (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow \{x \rightarrow 7, y \rightarrow 5\}
 \end{array}
 \end{array}$$



Let in Command

$$\frac{(E, m) \Downarrow v \quad (C, m[I \leftarrow v]) \Downarrow m'}{(\text{let } I = E \text{ in } C, m) \Downarrow m''}$$

Where $m''(y) = m'(y)$ for $y \neq I$ and
 $m''(I) = m(I)$ if $m(I)$ is defined,
and $m''(I)$ is undefined otherwise



Example

$$\frac{(x, \{x \rightarrow 5\}) \Downarrow 5 \quad (3, \{x \rightarrow 5\}) \Downarrow 3}{}$$

$$\frac{(x+3, \{x \rightarrow 5\}) \Downarrow 8}{}$$

$$\frac{(5, \{x \rightarrow 17\}) \Downarrow 5 \quad (x := x+3, \{x \rightarrow 5\}) \Downarrow \{x \rightarrow 8\}}{}$$

$$(\text{let } x = 5 \text{ in } (x := x+3), \{x \rightarrow 17\}) \Downarrow ?$$



Example

$$\frac{(x, \{x \rightarrow 5\}) \Downarrow 5 \quad (3, \{x \rightarrow 5\}) \Downarrow 3}{(x+3, \{x \rightarrow 5\}) \Downarrow 8}$$

$$\frac{(x+3, \{x \rightarrow 5\}) \Downarrow 8}{(5, \{x \rightarrow 17\}) \Downarrow 5 \quad (x := x+3, \{x \rightarrow 5\}) \Downarrow \{x \rightarrow 8\}}$$

$$\frac{(5, \{x \rightarrow 17\}) \Downarrow 5 \quad (x := x+3, \{x \rightarrow 5\}) \Downarrow \{x \rightarrow 8\}}{(\text{let } x = 5 \text{ in } (x := x+3), \{x \rightarrow 17\}) \Downarrow \{x \rightarrow 17\}}$$



Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explicitly*
- Clash of constructs apparent in awkward semantics



Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed



Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
 - Start with literals
 - Variables
 - Primitive operations
 - Evaluation of expressions
 - Evaluation of commands/declarations



Interpreter

- Takes abstract syntax trees as input
 - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
 - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next “state”
 - To get final value, put in a loop



Natural Semantics Example

- $\text{compute_exp} (\text{Var}(v), m) = \text{look_up } v \ m$
- $\text{compute_exp} (\text{Int}(n), _) = \text{Num } (n)$
- ...
- $\text{compute_com}(\text{IfExp}(b,c1,c2),m) =$
if $\text{compute_exp} (b,m) = \text{Bool}(\text{true})$
then $\text{compute_com} (c1,m)$
else $\text{compute_com} (c2,m)$



Natural Semantics Example

- $\text{compute_com}(\text{While}(b,c), m) =$
if $\text{compute_exp}(b,m) = \text{Bool}(\text{false})$
then m
else compute_com
 $(\text{While}(b,c), \text{compute_com}(c,m))$
- May fail to terminate - exceed stack limits
- Returns no useful information then



Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like
$$(C, m) \dashrightarrow (C', m') \quad \text{or} \quad (C, m) \dashrightarrow m'$$
- C, C' is code remaining to be executed
- m, m' represent the state/store/memory/environment
 - Partial mapping from identifiers to values
 - Sometimes m (or C) not needed
- Indicates exactly one step of computation



Expressions and Values

- C, C' used for commands; E, E' for expressions; U, V for values
- Special class of expressions designated as *values*
 - Eg 2, 3 are values, but $2+3$ is only an expression
- Memory only holds values
 - Other possibilities exist



Evaluation Semantics

- Transitions successfully stops when E/C is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence



Simple Imperative Programming Language

- $I \in \text{Identifiers}$
- $N \in \text{Numerals}$
- $B ::= \text{true} \mid \text{false} \mid B \ \& \ B \mid B \ \text{or} \ B \mid \text{not} \ B \mid E < E \mid E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- $C ::= \text{skip} \mid C; C \mid I ::= E \mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od}$



Transitions for Expressions

- Numerals are values
- Boolean values = {true, false}
- Identifiers: $(I, m) \dashrightarrow (m(I), m)$



Boolean Operations:

- Operators: (short-circuit)

$$\begin{array}{l} (\text{false} \ \& \ B, \ m) \ \rightarrow (\text{false}, m) \\ (\text{true} \ \& \ B, \ m) \ \rightarrow (B, m) \end{array} \quad \frac{(B, \ m) \ \rightarrow (B'', \ m)}{(B \ \& \ B', \ m) \ \rightarrow (B'' \ \& \ B', \ m)}$$

$$\begin{array}{l} (\text{true} \ \text{or} \ B, \ m) \ \rightarrow (\text{true}, m) \\ (\text{false} \ \text{or} \ B, \ m) \ \rightarrow (B, m) \end{array} \quad \frac{(B, \ m) \ \rightarrow (B'', \ m)}{(B \ \text{or} \ B', \ m) \ \rightarrow (B'' \ \text{or} \ B', \ m)}$$

$$\begin{array}{l} (\text{not} \ \text{true}, \ m) \ \rightarrow (\text{false}, m) \\ (\text{not} \ \text{false}, \ m) \ \rightarrow (\text{true}, m) \end{array} \quad \frac{(B, \ m) \ \rightarrow (B', \ m)}{(\text{not} \ B, \ m) \ \rightarrow (\text{not} \ B', \ m)}$$



Relations

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \sim E', m) \dashrightarrow (E'' \sim E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \sim E, m) \dashrightarrow (V \sim E', m)}$$

$(U \sim V, m) \dashrightarrow (\text{true}, m)$ or (false, m)
depending on whether $U \sim V$ holds or not



Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \text{ op } E', m) \dashrightarrow (E'' \text{ op } E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \text{ op } E, m) \dashrightarrow (V \text{ op } E', m)}$$

$(U \text{ op } V, m) \dashrightarrow (N, m)$ where N is the specified value for $U \text{ op } V$



Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory



Commands

$$(\text{skip}, m) \dashrightarrow m$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(I ::= E, m) \dashrightarrow (I ::= E', m)}$$

$$(I ::= V, m) \dashrightarrow m[I \leftarrow V]$$

$$\frac{(C, m) \dashrightarrow (C'', m')}{(C; C', m) \dashrightarrow (C''; C', m')} \quad \frac{(C, m) \dashrightarrow m'}{(C; C', m) \dashrightarrow (C', m')}$$



If Then Else Command - in English

- If the boolean guard in an `if_then_else` is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.



If Then Else Command

$(\text{if true then } C \text{ else } C' \text{ fi, } m) \dashrightarrow (C, m)$

$(\text{if false then } C \text{ else } C' \text{ fi, } m) \dashrightarrow (C', m)$

$$\frac{(B, m) \dashrightarrow (B', m)}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi, } m) \dashrightarrow (\text{if } B' \text{ then } C \text{ else } C' \text{ fi, } m)}$$



While Command

$(\text{while } B \text{ do } C \text{ od}, m) \dashrightarrow$
 $(\text{if } B \text{ then } C; \text{ while } B \text{ do } C \text{ od else skip fi}, m)$

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.



Example Evaluation

- First step:

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)
 $\rightarrow ?$



Example Evaluation

- First step:

$$(x > 5, \{x \rightarrow 7\}) \dashrightarrow ?$$

$$\begin{aligned} &(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ &\quad \{x \rightarrow 7\}) \\ &\quad \dashrightarrow ? \end{aligned}$$



Example Evaluation

- First step:

$$(x, \{x \rightarrow 7\}) \rightarrow (7, \{x \rightarrow 7\})$$

$$(x > 5, \{x \rightarrow 7\}) \rightarrow ?$$

$$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,}$$
$$\{x \rightarrow 7\})$$

$$\rightarrow ?$$



Example Evaluation

- First step:

$$(x, \{x \rightarrow 7\}) \rightarrow (7, \{x \rightarrow 7\})$$

$$\frac{(x > 5, \{x \rightarrow 7\}) \rightarrow (7 > 5, \{x \rightarrow 7\})}{(if\ x > 5\ then\ y := 2 + 3\ else\ y := 3 + 4\ fi,$$

$$\{x \rightarrow 7\})$$

$$\rightarrow ?$$



Example Evaluation

- First step:

$$(x, \{x \rightarrow 7\}) \dashrightarrow (7, \{x \rightarrow 7\})$$

$$\frac{(x > 5, \{x \rightarrow 7\}) \dashrightarrow (7 > 5, \{x \rightarrow 7\})}{(if\ x > 5\ then\ y := 2 + 3\ else\ y := 3 + 4\ fi,$$

$$\{x \rightarrow 7\})$$

$$\dashrightarrow (if\ 7 > 5\ then\ y := 2 + 3\ else\ y := 3 + 4\ fi,$$
$$\{x \rightarrow 7\})$$



Example Evaluation

- Second Step:

$$\frac{(7 > 5, \{x \rightarrow 7\}) \rightarrow (\text{true}, \{x \rightarrow 7\})}{(\text{if } 7 > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\})}$$
$$\rightarrow (\text{if true then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\})$$

- Third Step:

$$(\text{if true then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\})$$
$$\rightarrow (y := 2 + 3, \{x \rightarrow 7\})$$



Example Evaluation

- Fourth Step:

$$\frac{(2+3, \{x \rightarrow 7\}) \rightarrow (5, \{x \rightarrow 7\})}{(y := 2+3, \{x \rightarrow 7\}) \rightarrow (y := 5, \{x \rightarrow 7\})}$$

- Fifth Step:

$$(y := 5, \{x \rightarrow 7\}) \rightarrow \{y \rightarrow 5, x \rightarrow 7\}$$



Example Evaluation

- Bottom Line:

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)

--> (if $7 > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)

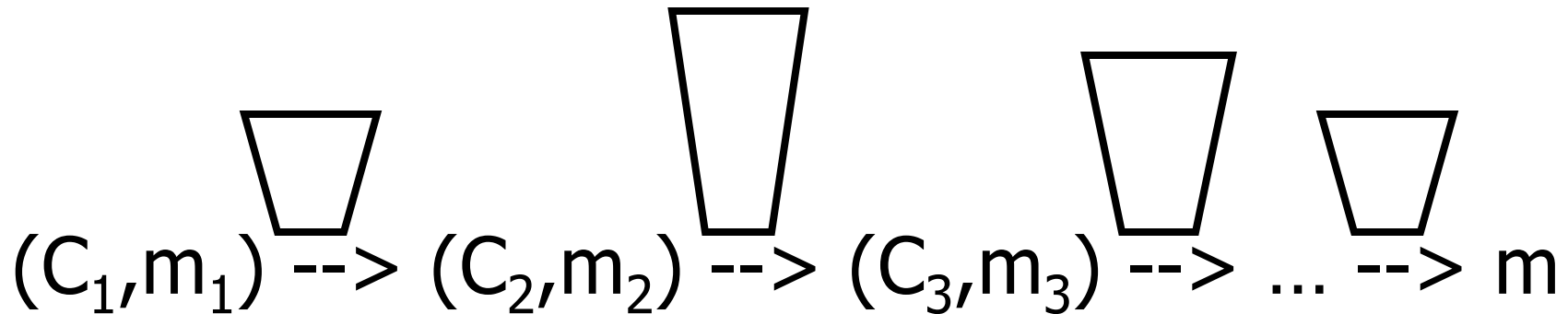
--> (if true then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}$)

--> ($y := 2 + 3$, $\{x \rightarrow 7\}$)

--> ($y := 5$, $\{x \rightarrow 7\}$) --> $\{y \rightarrow 5, x \rightarrow 7\}$

Transition Semantics Evaluation

- A sequence of steps with trees of justification for each step



- Let \dashrightarrow^* be the transitive closure of \dashrightarrow
- Ie, the smallest transitive relation containing \dashrightarrow



Adding Local Declarations

- Add to expressions:
- $E ::= \dots \mid \text{let } I = E \text{ in } E' \mid \text{fun } I \rightarrow E \mid E E'$
- $\text{fun } I \rightarrow E$ is a value
- Could handle local binding using state, but have assumption that evaluating expressions doesn't alter the environment
- We will use substitution here instead
- **Notation:** $E[E' / I]$ means replace all free occurrence of I by E' in E



Call-by-value (Eager Evaluation)

$$(\text{let } I = V \text{ in } E, m) \rightarrow (E[V/I], m)$$

$$(E, m) \rightarrow (E'', m)$$

$$\frac{(E, m) \rightarrow (E'', m)}{(\text{let } I = E \text{ in } E', m) \rightarrow (\text{let } I = E'' \text{ in } E')}$$

$$((\text{fun } I \rightarrow E) V, m) \rightarrow (E[V/I], m)$$

$$(E', m) \rightarrow (E'', m)$$

$$\frac{(E', m) \rightarrow (E'', m)}{((\text{fun } I \rightarrow E) E', m) \rightarrow ((\text{fun } I \rightarrow E) E'', m)}$$

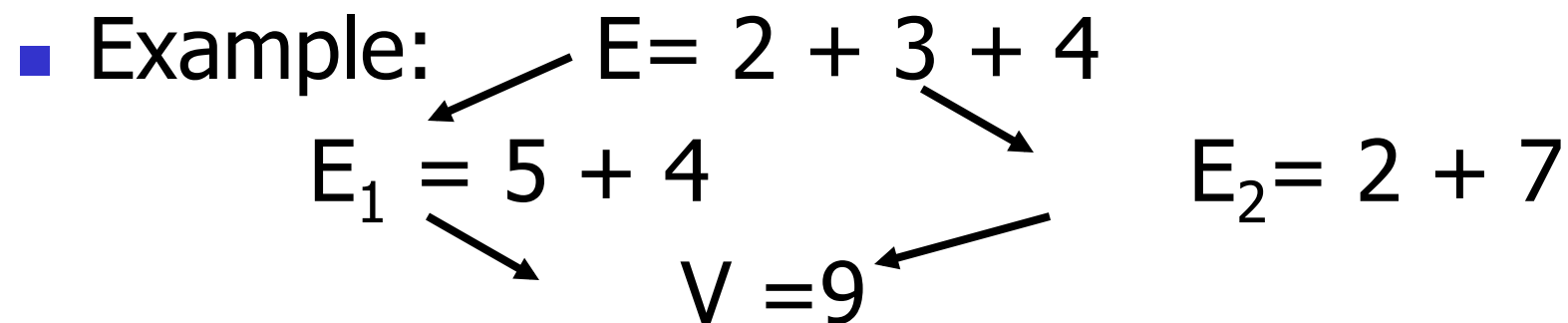


Call-by-name (Lazy Evaluation)

- $(\text{let } I = E \text{ in } E', m) \dashrightarrow (E' [E / I], m)$
- $((\text{fun } I \rightarrow E') E, m) \dashrightarrow (E' [E / I], m)$
- Question: Does it make a difference?
- It can depending on the language

Church-Rosser Property

- Church-Rosser Property: If $E \rightarrow^* E_1$ and $E \rightarrow^* E_2$, if there exists a value V such that $E_1 \rightarrow^* V$, then $E_2 \rightarrow^* V$
- Also called **confluence** or **diamond property**





Does It always Hold?

- No. Languages with side-effects tend not be Church-Rosser with the combination of call-by-name and call-by-value
- Alonzo Church and Barkley Rosser proved in 1936 the λ -calculus does have it
- Benefit of Church-Rosser: can check equality of terms by evaluating them (Given evaluation strategy might not terminate, though)