Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Sample Grammar

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

<term> ::= <factor> | <factor> * <term>
| <factor> / <term>

<factor> ::= <id> | (<expr>)

Tokens as OCaml Types

- + - * / ( ) <id>
- Becomes an OCaml datatype

```
type token =
    Id_token of string
  | Left_parenthesis | Right_parenthesis
  | Times_token | Divide_token
  | Plus_token | Minus_token
```


Parse Trees as Datatypes

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

```
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
```


Parse Trees as Datatypes

<term> ::= <factor> | <factor> * <term>
| <factor> / <term>

```
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
```


Parse Trees as Datatypes

<factor> ::= <id> | (<expr>)

```
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

Parsing Lists of Tokens

- Will create three mutually recursive functions:
  - expr : token list -> (expr * token list)
  - term : token list -> (term * token list)
  - factor : token list -> (factor * token list)
- Each parses what it can and gives back parse and remaining tokens

Parsing an Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
    (match tokens_after_term
     with( Plus_token  :: tokens_after_plus) ->
```


Parsing an Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
    (match tokens_after_term
     with ( Plus_token  :: tokens_after_plus) ->
```


Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
    (match tokens_after_term
     with ( Plus_token  :: tokens_after_plus) ->
```


Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
    (match tokens_after_term
     with ( Plus_token  :: tokens_after_plus) ->
```


Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
    (match tokens_after_term
     with ( Plus_token  :: tokens_after_plus) ->
```

Parsing a Plus Expression

<expr> ::= <term> + <expr>

```
(match expr tokens_after_plus
  with ( expr_parse , tokens_after_expr) ->
  ( Plus_Expr ( term_parse , expr_parse ),
   tokens_after_expr))
```


Parsing a Plus Expression

<expr> ::= <term> + <expr>

```
(match expr tokens_after_plus
  with ( expr_parse , tokens_after_expr) ->
  ( Plus_Expr ( term_parse , expr_parse ),
   tokens_after_expr))
```


Building Plus Expression Parse Tree

<expr> ::= <term> + <expr>

```
(match expr tokens_after_plus
  with ( expr_parse , tokens_after_expr) ->
  ( Plus_Expr ( term_parse , expr_parse ),
   tokens_after_expr))
```


Parsing a Minus Expression

<expr> ::= <term> - <expr>

```
| ( Minus_token :: tokens_after_minus) ->
  (match expr tokens_after_minus
  with ( expr_parse , tokens_after_expr) ->
 ( Minus_Expr ( term_parse , expr_parse ),
  tokens_after_expr))
```


Parsing a Minus Expression

<expr> ::= <term> - <expr>

```
| ( Minus_token :: tokens_after_minus) ->
  (match expr tokens_after_minus
  with ( expr_parse , tokens_after_expr) ->
 ( Minus_Expr ( term_parse , expr_parse ),
  tokens_after_expr))
```


Parsing an Expression as a Term

<expr> ::= <term>

```
| _ -> (Term_as_Expr term_parse ,
  tokens_after_term)))
```

- Code for **term** is same except for replacing addition with multiplication and subtraction with division

Parsing Factor as Id

<factor> ::= <id>

```
and factor  tokens =
 (match tokens
  with (Id_token id_name :: tokens_after_id) =
   ( Id_as_Factor id_name, tokens_after_id)
```


Parsing Factor as Parenthesized Expression

<factor> ::= (<expr>)

```
| factor ( Left_parenthesis :: tokens) =
    (match expr tokens
     with ( expr_parse , tokens_after_expr) ->
```


Parsing Factor as Parenthesized Expression

<factor> ::= (<expr>)

```
(match tokens_after_expr
with Right_parenthesis :: tokens_after_rparen ->
 ( Parenthesized_Expr_as_Factor expr_parse ,
  tokens_after_rparen)
```


Error Cases

- What if no matching right parenthesis?

  | _ -> raise (Failure "No matching rparen")))

- What if no leading id or left parenthesis?

  | _ -> raise (Failure "No id or lparen"));;

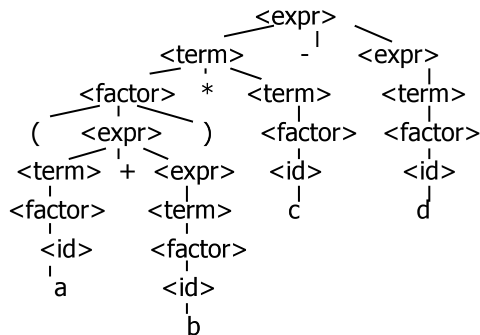
(a + b) * c - d

```
expr [Left_parenthesis; Id_token "a”;
  Plus_token; Id_token "b”;
  Right_parenthesis; Times_token;
  Id_token "c”; Minus_token;
  Id_token "d"];;
```


(a + b) * c - d

```
- : expr * token list =
(Minus_Expr
 (Mult_Term
  (Parenthesized_Expr_as_Factor
   (Plus_Expr
    (Factor_as_Term (Id_as_Factor "a"),
     Term_as_Expr (Factor_as_Term
  (Id_as_Factor "b")))),
   Factor_as_Term (Id_as_Factor "c")),
  Term_as_Expr (Factor_as_Term (Id_as_Factor
  "d"))),
 [])
```

(a + b) * c – d

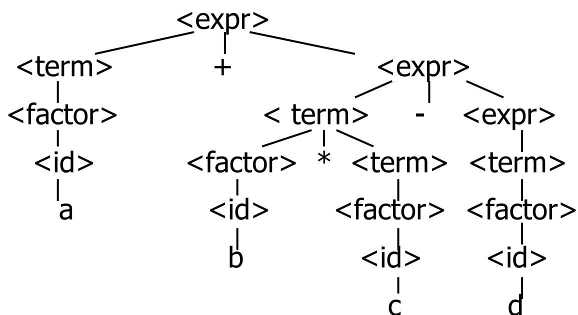


a + b * c – d

\# expr [Id_token "a"; Plus_token; Id_token "b"; Times_token; Id_token "c"; Minus_token; Id_token "d"];;

```
- : expr * token list =
(Plus_Expr
 (Factor_as_Term (Id_as_Factor "a"),
  Minus_Expr
   (Mult_Term (Id_as_Factor "b", Factor_as_Term
  (Id_as_Factor "c")),
    Term_as_Expr (Factor_as_Term (Id_as_Factor
  "d")))),
 [])
```


a + b * c – d



(a + b * c - d

\# expr [Left_parenthesis; Id_token "a"; Plus_token; Id_token "b"; Times_token; Id_token "c"; Minus_token; Id_token "d"];;

Exception: Failure "No matching rparen".

Can't parse because it was expecting a right parenthesis but it got to the end without finding one

a + b) * c - d *)

expr [Id_token "a"; Plus_token; Id_token "b"; Right_parenthesis; Times_token; Id_token "c"; Minus_token; Id_token "d"];;

```
- : expr * token list =
(Plus_Expr
 (Factor_as_Term (Id_as_Factor "a"),
  Term_as_Expr (Factor_as_Term (Id_as_Factor
  "b"))),
 [Right_parenthesis; Times_token; Id_token "c";
  Minus_token; Id_token "d"])
```


Parsing Whole String

- Q: How to guarantee whole string parses?
- A: Check returned tokens empty

```
let parse tokens =
   match expr tokens
    with (expr_parse, []) -> expr_parse
    | _ -> raise (Failure "No parse");;
```

- Fixes <expr> as start symbol

Streams in Place of Lists

- More realistically, we don't want to create the entire list of tokens before we can start parsing
- We want to generate one token at a time and use it to make one step in parsing
- Will use (token * (unit -> token)) or (token * (unit -> token option))

  in place of token list

Problems for Recursive-Descent Parsing

- Left Recursion:

  A ::= Aw

  translates to a subroutine that loops forever
- Indirect Left Recursion:

  A ::= Bw

  B ::= Av

  causes the same problem

Problems for Recursive-Descent Parsing

- Parser must always be able to choose the next action based only only the very next token
- Pairwise Disjointedness Test: Can we always determine which rule (in the non-extended BNF) to choose based on just the first token

Pairwise Disjointedness Test

- For each rule

  A ::= *y*

Calculate

FIRST (*y*) =

{a | *y* =>* a*w*} ∪ {ε | if y =>* ε}

- For each pair of rules A ::= *y* and A ::= *z*, require FIRST(*y*) ∩ FIRST(*z*) = { }

Example

Grammar:

<S> ::= <A> a <B> b

<A> ::= <A> b | b

<B> ::= a <B> | a

FIRST (<A> b) = {b}

FIRST (b) = {b}

Rules for <A> not pairwise disjoint

Eliminating Left Recursion

- Rewrite grammar to shift left recursion to right recursion
  - Changes associativity
- Given

<expr> ::= <expr> + <term> and

<expr> ::= <term>

- Add new non-terminal <e> and replace above rules with

<expr> ::= <term><e>

<e> ::= + <term><e> | ε

Factoring Grammar

- Test too strong: Can't handle

  <expr> ::= <term> [(+ | -) <expr>]
- Answer: Add new non-terminal and replace above rules by

  <expr> ::= <term><e>

  <e> ::= + <term><e>

  <e> ::= - <term><e>

  <e> ::= ε
- You are delaying the decision point

Example

Both <A> and <B> have problems:

<S> ::= <A> a <B> b

<A> ::= <A> b | b

<B> ::= a <B> | a

Transform grammar to:

<S> ::= <A> a <B> b

<A> ::-= b<A1>

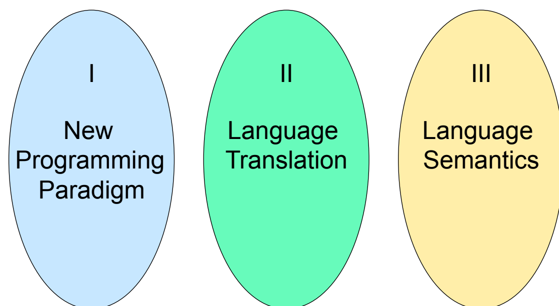
<A1> :: b<A1> | ε

<B> ::= a<B1>

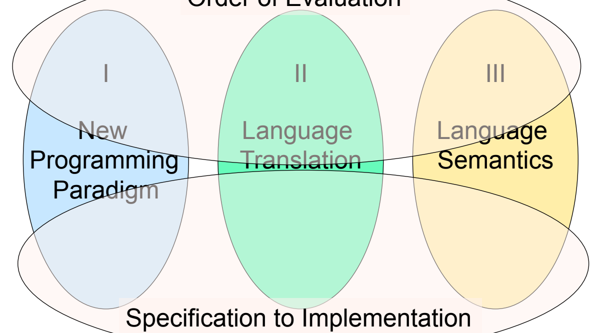
<B1> ::= a<B1> | ε

Programming Languages & Compilers

Three Main Topics of the Course

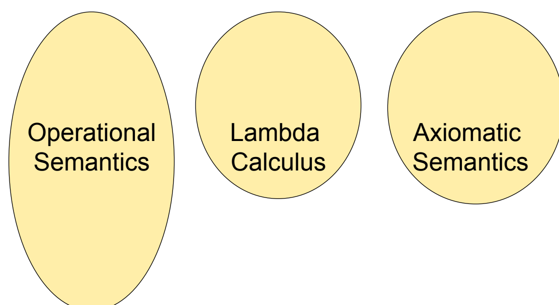


Programming Languages & Compilers

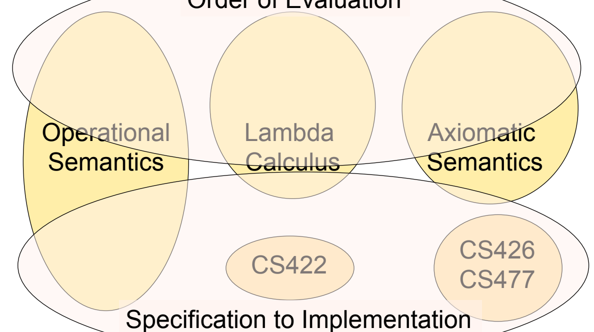


Programming Languages & Compilers

III : Language Semantics



Programming Languages & Compilers



Semantics

- Expresses the meaning of syntax
- Static semantics
  - Meaning based only on the form of the expression without executing it
  - Usually restricted to type checking / type inference

Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
  - Operational Semantics
  - Axiomatic Semantics
  - Denotational Semantics

Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes

Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations

Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages

Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :

  {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*

Denotational Semantics

- Construct a function $\mathcal{M}$ assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs

Natural Semantics

- Aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- E::= N | I | $E + E$ | $E * E$ | $E - E$ | $- E$
- C::= skip | C;C | I ::= E | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: $(\text{true},m) \Downarrow \text{true}$

  $(\text{false},m) \Downarrow \text{false}$

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \,\&\, B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \,\&\, B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation $\sim$ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U\ op\ V = N}{(E\ op\ E', m) \Downarrow N}$$

where N is the specified value for $U\ op\ V$

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment: $$\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I \text{ <-- } V]}$$

Sequencing: $$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \ (C,m)\Downarrow m' \ (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

$$\frac{}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: If Then Else Rule

$$\frac{\dfrac{}{(x > 5, \{x \text{ -> } 7\})\Downarrow ?}}{(\text{if x > 5 then y:= 2 + 3 else y:=3 + 4 fi}, \{x \text{ -> } 7\}) \Downarrow ?}$$

Example: Arith Relation

? > ? = ?

(x,{x->7})⇓? (5,{x->7})⇓?

(x > 5, {x -> 7})⇓?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Identifier(s)

7 > 5 = true

(x,{x->7})⇓7 (5,{x->7})⇓5

(x > 5, {x -> 7})⇓?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Arith Relation

7 > 5 = true

(x,{x->7})⇓7 (5,{x->7})⇓5

(x > 5, {x -> 7})⇓true

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: If Then Else Rule

7 > 5 = true

(x,{x->7})⇓7 (5,{x->7})⇓5

(x > 5, {x -> 7})⇓true   (y:= 2 + 3,{x-> 7}) ⇓ ? .

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Assignment

7 > 5 = true   (2+3, {x->7})⇓?

(x,{x->7})⇓7 (5,{x->7})⇓5   (y:= 2 + 3,{x-> 7})

(x > 5, {x -> 7})⇓true   ⇓ ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Arith Op

? + ? = ?

(2,{x->7})⇓? (3,{x->7}) ⇓?

7 > 5 = true   (2+3, {x->7})⇓?

(x,{x->7})⇓7 (5,{x->7})⇓5   (y:= 2 + 3,{x-> 7})

(x > 5, {x -> 7})⇓true   ⇓? .

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Numerals

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

7 > 5 = true (2+3, {x->7})⇓?

(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})

(x > 5, {x -> 7})⇓true ⇓ ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Arith Op

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

7 > 5 = true (2+3, {x->7})⇓5

(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})

(x > 5, {x -> 7})⇓true ⇓?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Assignment

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

7 > 5 = true (2+3, {x->7})⇓5

(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})

(x > 5, {x -> 7})⇓true ⇓ {x->7, y->5}

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓?

Example: If Then Else Rule

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

7 > 5 = true (2+3, {x->7})⇓5

(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3,{x-> 7})

(x > 5, {x -> 7})⇓true ⇓ {x->7, y->5}

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ {x->7, y->5}

Let in Command

(*E*,*m*) ⇓v (*C*,*m*[*I*<-*v*]) ⇓ *m'*

(let *I* = *E* in *C*, *m*) ⇓ *m''*

Where *m''* (*y*) = *m'* (*y*) for *y*≠ *I* and *m''* (*I*) = *m* (*I*) if *m*(*I*) is defined, and *m''* (*I*) is undefined otherwise

Example

(x,{x->5}) ⇓ 5 (3,{x->5}) ⇓ 3

(x+3,{x->5}) ⇓ 8

(5,{x->17}) ⇓ 5 (x:=x+3,{x->5}) ⇓ {x->8}

(let x = 5 in (x:=x+3), {x -> 17}) ⇓ ?

Example

$$\frac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{}$$

$$\frac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad (x:=x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}{(\text{let } x = 5 \text{ in } (x:=x+3), \{x \text{->} 17\}) \Downarrow \{x\text{->}17\}}$$

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =

  if compute_exp (b,m) = Bool(true)

  then compute_com (c1,m)

  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =
  if compute_exp (b,m) = Bool(false)
  then m
  else compute_com
  (While(b,c), compute_com(c,m))

- May fail to terminate - exceed stack limits
- Returns no useful information then

Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like
  (C, *m*) --> (C', *m'*) or (C,*m*) --> *m'*
- *C, C'* is code remaining to be executed
- m, m' represent the state/store/memory/ environment
  - Partial mapping from identifiers to values
  - Sometimes *m* (or *C*) not needed
- Indicates exactly one step of computation

Expressions and Values

- *C, C'* used for commands; *E, E' for* expressions; *U,V* for values
- Special class of expressions designated as *values*
  - Eg 2, 3 are values, but 2+3 is only an expression
- Memory only holds values
  - Other possibilities exist

Evaluation Semantics

- Transitions successfully stops when E/*C* is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

Simple Imperative Programming Language

- $I \in$ *Identifiers*
- $N \in$ *Numerals*
- *B* ::= true | false | *B* & *B* | *B* or *B* | not *B* | *E* < *E* | *E* = *E*
- *E::= N | I | E + E | E * E | E - E | - E*
- C::= skip | *C*;*C* | *I* ::= *E*
  | if *B* then *C* else *C* fi | while *B* do *C* od

Transitions for Expressions

- Numerals are values
- Boolean values = {true, false}
- Identifiers: (*I*,*m*) --> (*m*(*I*), *m*)

Boolean Operations:

- Operators: (short-circuit)

(false & B, m) --> (false,m)

(true & B, m) --> (B,m)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \,\&\, B', m) \dashrightarrow (B'' \,\&\, B', m)}$$

(true or B, m) --> (true,m)

(false or B, m) --> (B,m)

$$\frac{(B, m) \dashrightarrow (B'', m)}{(B \text{ or } B', m) \dashrightarrow (B'' \text{ or } B', m)}$$

(not true, m) --> (false,m)

(not false, m) --> (true,m)

$$\frac{(B, m) \dashrightarrow (B', m)}{(\text{not } B, m) \dashrightarrow (\text{not } B', m)}$$

Relations

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \sim E', m) \dashrightarrow (E'' \sim E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \sim E, m) \dashrightarrow (V \sim E', m)}$$

($U \sim V$, m) --> (true,m) or (false,m) depending on whether $U \sim V$ holds or not

Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \text{ op } E', m) \dashrightarrow (E'' \text{ op } E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \text{ op } E, m) \dashrightarrow (V \text{ op } E', m)}$$

(U op V, m) --> (N,m) where N is the specified value for U op V

Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory

Commands

(skip, m) --> m

$$\frac{(E, m) \dashrightarrow (E', m)}{(I ::= E, m) \dashrightarrow (I ::= E', m)}$$

(I::=V,m) --> $m[I \leftarrow V]$

$$\frac{(C, m) \dashrightarrow (C'', m')}{(C;C', m) \dashrightarrow (C'';C', m')} \qquad \frac{(C, m) \dashrightarrow m'}{(C;C', m) \dashrightarrow (C', m')}$$

If Then Else Command - in English

- If the boolean guard in an if_then_else is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.

If Then Else Command

(if true then C else C' fi, m) --> (C, m)

(if false then C else C' fi, m) --> (C', m)

$$\frac{(B,m) \dashrightarrow (B',m)}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \dashrightarrow (\text{if } B' \text{ then } C \text{ else } C' \text{ fi}, m)}$$

While Command

(while B do C od, m) -->
(if B then C; while B do C od else skip fi, m)

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.