

## Recursion over Recursive Data Types

\# type exp = VarExp of string | ConstExp of const
| BinOpAppExp of bin_op * exp * exp
| FunExp of string * exp | AppExp of exp * exp

- How to count the number of variables in an exp?
\# let rec varCnt exp = match exp with VarExp x ->
| ConstExp c ->
| BinOpAppExp (b, e1, e2) ->
| FunExp (x,e) ->
| AppExp (e1, e2) ->


## Your turn now

## Try Problem 3 on MP3

## Recursion over Recursive Data Types

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| BinOpAppExp of bin_op * exp * exp
| FunExp of string * $\exp \mid$ AppExp of $\exp * \exp$

- How to count the number of variables in an exp?


## Recursion over Recursive Data Types

\# type exp = VarExp of string | ConstExp of const | BinOpAppExp of bin_op * exp * exp
| FunExp of string * exp | AppExp of exp * exp

- How to count the number of variables in an exp?
\# let rec varCnt exp =
match exp with VarExp $x->1$
| ConstExp c -> 0
| BinOpAppExp (b, e1, e2) -> varCnt e1 + varCnt e2
| FunExp (x,e) -> 1 + varCnt e
| AppExp (e1, e2) -> varCnt e1 + varCnt e2

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## Mutually Recursive Types

\# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree
| More of ('a tree * 'a treeList);;
type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList)

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Mutually Recursive Types - Values
\# let tree =
TreeNode
(More (TreeLeaf 5,
(More (TreeNode
(More (TreeLeaf 3,
Last (TreeLeaf 2))),
Last (TreeLeaf 7)))));;

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Mutually Recursive Types - Values


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## Mutually Recursive Functions

```
# let rec fringe tree =
    match tree with (TreeLeaf x) -> [x]
    | (TreeNode list) -> list_fringe list
    and list_fringe tree_list =
        match tree_list with (Last tree) -> fringe tree
    | (More (tree,list)) ->
        (fringe tree) @ (list_fringe list);;
```

    val fringe : 'a tree -> 'a list = <fun>
    val list_fringe : 'a treeList -> 'a list = <fun>
    Mutually Recursive Types - Values
val tree : int tree =
TreeNode
(More
(TreeLeaf 5,
More
(TreeNode (More (TreeLeaf 3, Last (TreeLeaf 2))), Last (TreeLeaf 7))))

Mutually Recursive Types - Values
A more conventional picture


## Mutually Recursive Functions

\# fringe tree;;

- : int list = [5; 3; 2; 7]


## Problem

\# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size

## Problem

\# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size
let rec tree_size $t=$
match t with TreeLeaf _ -> 1
| TreeNode ts -> treeList_size ts

## Problem

\# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size let rec tree_size $\mathrm{t}=$ match t with TreeLeaf _ ->
| TreeNode ts ->

## Problem

\# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size and treeList_size let rec tree_size $\mathrm{t}=$
match $t$ with TreeLeaf _ -> 1
| TreeNode ts -> treeList_size ts and treeList_size ts =

## Problem

\# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size and treeList_size
let rec tree_size $t=$
match t with TreeLeaf _ -> 1
| TreeNode ts -> treeList_size ts and treeList_size ts =
match ts with Last t-> tree_size t
| More t ts' -> tree_size t + treeList_size ts'

## Problem

\# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
Define tree_size and treeList_size
let rec tree_size $t=$
match t with TreeLeaf _ -> 1
| TreeNode ts -> treeList_size ts
and treeList_size ts =
match ts with Last t -> tree_size t
| More t ts' -> tree_size t + treeList_size ts'

## Nested Recursive Type Values

\# let ltree =
TreeNode(5,
[TreeNode (3, []);
TreeNode (2, [TreeNode (1, []);
TreeNode (7, [])]);
TreeNode (5, [])]);;

## Nested Recursive Types

\# type 'a labeled_tree =
TreeNode of ('a * 'a labeled_tree list);;
type 'a labeled_tree = TreeNode of ('a * 'a labeled_tree list)

Nested Recursive Type Values
val Itree : int labeled_tree = TreeNode
(5,
[TreeNode (3, []); TreeNode (2, [TreeNode (1, []); TreeNode (7, [])]); TreeNode (5, [])])

## Nested Recursive Type Values



## Mutually Recursive Functions

\＃let rec flatten＿tree labtree＝ match labtree with TreeNode（x，treelist）
－＞x：：flatten＿tree＿list treelist
and flatten＿tree＿list treelist＝
match treelist with［］－＞［］
｜labtree：：labtrees
－＞flatten＿tree labtree
＠flatten＿tree＿list labtrees；；

## Infinite Recursive Values

\＃let rec ones＝1：：ones；；
val ones ：int list＝
$[1 ; 1 ; 1 ; 1 ; \ldots]$
\＃match ones with $\mathrm{x}:$ ：＿－＞ x ；；
Characters 0－25：
Warning：this pattern－matching is not exhaustive．
Here is an example of a value that is not matched：
［］
match ones with $\mathrm{x}:$ ：＿－＞ x ；；
ヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘヘ
$-:$ int＝ 1

## Infinite Recursive Values

\＃match lab＿tree
with TreeNode（x，＿）－＞x；；
－：int＝ 2

## Mutually Recursive Functions

val flatten＿tree ：＇a labeled＿tree－＞＇a list＝ ＜fun＞
val flatten＿tree＿list ：＇a labeled＿tree list－＞＇a list $=$＜fun＞
\＃flatten＿tree Itree；；
－：int list＝［5；3；2；1；7；5］
－Nested recursive types lead to mutually recursive functions

## Infinite Recursive Values

\＃let rec lab＿tree＝TreeNode（2，tree＿list） and tree＿list＝［lab＿tree；lab＿tree］；； val lab＿tree ：int labeled＿tree＝
TreeNode（2，［TreeNode（．．．）；TreeNode（．．．）］）
val tree＿list ：int labeled＿tree list＝
［TreeNode（2，［TreeNode（．．．）；
TreeNode（．．．）］）；
TreeNode（2，［TreeNode（．．．）；
TreeNode（．．．）］）］
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## Records

－Records serve the same programming purpose as tuples
－Provide better documentation，more readable code
－Allow components to be accessed by label instead of position
－Labels（aka field names must be unique）
－Fields accessed by suffix dot notation

## Record Types

- Record types must be declared before they can be used in OCaml
\# type person = \{name : string; ss : (int * int * int); age : int $\}$;;
type person = \{ name : string; ss : int * int * int; age : int; \}
- person is the type being introduced
- name, ss and age are the labels, or fields


## Record Pattern Matching

\# let $\{$ name $=$ elsa; age = age; ss = (_,_s3) \} = teacher;;
val elsa : string = "Elsa L. Gunter"
val age : int = 102
val s3 : int = 6244

## Record Values

\# let student = \{ss=(325,40,1276); name="Joseph Martins"; age=22\};;
val student : person =
\{name = "Joseph Martins"; ss = (325, 40, 1276); age $=22\}$
\# student = teacher;;

- : bool = false


## Record Values

## Records built with labels; order does not matter

\# let teacher $=$ \{name = "Elsa L. Gunter"; age = 102; ss = (119,73,6244) $\}$;;
val teacher : person =
\{name = "Elsa L. Gunter"; ss = (119, 73, 6244); age $=102\}$

## Record Field Access

\# let soc_sec = teacher.ss;; val soc_sec : int * int * int = (119, $73,6244)$

## New Records from Old

\# let birthday person = \{person with age = person.age + 1\};;
val birthday : person -> person = <fun> \# birthday teacher;;

- : person = \{name = "Elsa L. Gunter"; ss = (119, 73, 6244); age $=103\}$


## New Records from Old

\# let new_id name soc_sec person = \{person with name = name; ss = soc_sec\};; val new_id : string -> int * int * int -> person -> person = <fun>
\# new_id "Guieseppe Martin" $(523,04,6712)$
student;;

- : person = \{name = "Guieseppe Martin"; ss $=(523,4,6712) ;$ age $=22\}$

