Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

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Terms

- A function is in Direct Style when it returns its result back to the caller.
- A Tail Call occurs when a function returns the result of another function call without any more computations (eg tail recursion)
- A function is in Continuation Passing Style when it, and every function call in it, passes its result to another function.
- Instead of returning the result to the caller, we pass it forward to another function.

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Terminology

- Tail Position: A subexpression s of expressions e, such that if evaluated, will be taken as the value of e
  - if (x>3) then x + 2 else x - 4
  - let x = 5 in x + 4
- Tail Call: A function call that occurs in tail position
  - if (h x) then f x else (x + g x)

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Terminology

- Available: A function call that can be executed by the current expression
- The fastest way to be unavailable is to be guarded by an abstraction (anonymous function, lambda lifted).
  - if (h x) then f x else (x + g x)
  - if (h x) then (fun x -> f x) else (g (x + x))

                                        Not available

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CPS Transformation

- Step 1: Add continuation argument to any function definition:
  - let f arg = e $\Rightarrow$ let f arg k = e
  - Idea: Every function takes an extra parameter saying where the result goes
- Step 2: A simple expression in tail position should be passed to a continuation instead of returned:
  - return a $\Rightarrow$ k a
  - Assuming a is a constant or variable.
  - "Simple" = "No available function calls."

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CPS Transformation

- Step 3: Pass the current continuation to every function call in tail position
  - return f arg $\Rightarrow$ f arg k
  - The function "isn't going to return," so we need to tell it where to put the result.

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CPS Transformation

- Step 4: Each function call not in tail position needs to be converted to take a new continuation (containing the old continuation as appropriate)
  - return op (f arg) $\Rightarrow$ f arg (fun r -> k(op r))
  - op represents a primitive operation

  - return f(g arg) $\Rightarrow$ g arg (fun r-> f r k)

Example

Before:

```
let rec add_list lst =
match lst with
  [ ] -> 0
| 0 :: xs -> add_list xs
| x :: xs -> (+) x
   (add_list xs);;
```

After:

```
let rec add_listk lst k =
          (* rule 1 *)
match lst with
| [ ] -> k 0 (* rule 2 *)
| 0 :: xs -> add_listk xs k
          (* rule 3 *)
| x :: xs -> add_listk xs
       (fun r -> k ((+) x r));;
          (* rule 4 *)
```
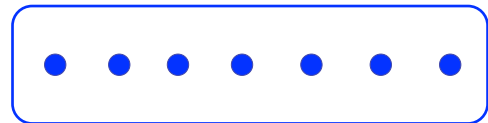

Variants - Syntax (slightly simplified)

- type *name* = C_1 [of ty_1] | . . . | C_n [of ty_n]
- Introduce a type called *name*
- (fun x -> C_i x) : ty_1 -> *name*
- C_i is called a *constructor*; if the optional type argument is omitted, it is called a *constant*
- Constructors are the basis of almost all pattern matching

Enumeration Types as Variants

An enumeration type is a collection of distinct values

In C and Ocaml they have an order structure; order by order of input

---

Enumeration Types as Variants

```
# type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;
type weekday =
   Monday
  | Tuesday
  | Wednesday
  | Thursday
  | Friday
  | Saturday
  | Sunday
```

Functions over Enumerations

```
# let day_after day = match day with
    Monday -> Tuesday
  | Tuesday -> Wednesday
  | Wednesday -> Thursday
  | Thursday -> Friday
  | Friday -> Saturday
  | Saturday -> Sunday
  | Sunday -> Monday;;
val day_after : weekday -> weekday = <fun>
```

Functions over Enumerations

```
# let rec days_later n day =
    match n with 0 -> day
    | _ -> if n > 0
        then day_after (days_later (n - 1) day)
        else days_later (n + 7) day;;
val days_later : int -> weekday -> weekday
  = <fun>
```

Functions over Enumerations

```
# days_later 2 Tuesday;;
- : weekday = Thursday
# days_later (-1) Wednesday;;
- : weekday = Tuesday
# days_later (-4) Monday;;
- : weekday = Thursday
```

Problem:

```
# type weekday = Monday | Tuesday |
  Wednesday
  | Thursday | Friday | Saturday | Sunday;;
```
- Write function is_weekend : weekday -> bool
```
let is_weekend day =
```

Problem:

```
# type weekday = Monday | Tuesday |
  Wednesday
  | Thursday | Friday | Saturday | Sunday;;
```
- Write function is_weekend : weekday -> bool
```
let is_weekend day =
    match day with Saturday -> true
    | Sunday -> true
    | _ -> false
```

Example Enumeration Types
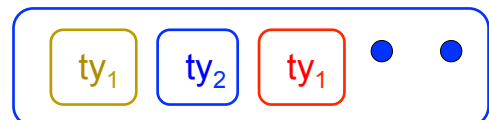
```
# type bin_op = IntPlusOp |  IntMinusOp
        | EqOp | CommaOp | ConsOp

# type mon_op = HdOp | TlOp | FstOp
        | SndOp
```

Disjoint Union Types

- Disjoint union of types, with some possibly occurring more than once

- We can also add in some new singleton elements

Disjoint Union Types

```
# type id = DriversLicense of int
  | SocialSecurity of int | Name of string;;
type id = DriversLicense of int | SocialSecurity
  of int | Name of string
# let check_id id = match id with
    DriversLicense num ->
      not (List.mem num [13570; 99999])
    | SocialSecurity num -> num < 900000000
    | Name str -> not (str = "John Doe");;
val check_id : id -> bool = <fun>
```

Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

```
type currency =
    Dollar of int
  | Pound of int
  | Euro of int
  | Yen of int
```

Example Disjoint Union Type

```
# type const =
    BoolConst of bool
  | IntConst of int
  | FloatConst of float
  | StringConst of string
  | NilConst
  | UnitConst
```

Example Disjoint Union Type

```
# type const = BoolConst of bool
    | IntConst of int | FloatConst of float
    | StringConst of string  | NilConst
    | UnitConst
```

- How to represent 7 as a const?
- Answer: IntConst 7

Polymorphism in Variants

- The type 'a option is gives us something to represent non-existence or failure

```
# type 'a option = Some of 'a | None;;
type 'a option = Some of 'a | None
```

- Used to encode partial functions
- Often can replace the raising of an exception

Functions producing option

```
# let rec first p list =
    match list with [ ] -> None
    | (x::xs) -> if p x then Some x else first p xs;;
val first : ('a -> bool) -> 'a list -> 'a option = <fun>
# first (fun x -> x > 3) [1;3;4;2;5];;
- : int option = Some 4
# first (fun x -> x > 5) [1;3;4;2;5];;
- : int option = None
```

Functions over option

```
# let result_ok r =
    match r with None -> false
    | Some _ -> true;;
val result_ok : 'a option -> bool = <fun>
# result_ok (first (fun x -> x > 3) [1;3;4;2;5]);;
- : bool = true
# result_ok (first (fun x -> x > 5) [1;3;4;2;5]);;
- : bool = false
```

Problem

- Write a hd and tl on lists that doesn't raise an exception and works at all types of lists.

Problem

- Write a hd and tl on lists that doesn't raise an exception and works at all types of lists.

- let hd list =
      match list with [] -> None
        | (x::xs) -> Some x
- let tl list =
      match list with [] -> None
        | (x::xs) -> Some xs

Mapping over Variants

```
# let optionMap f opt =
    match opt with None -> None
    | Some x -> Some (f x);;
val optionMap : ('a -> 'b) -> 'a option -> 'b
  option = <fun>
# optionMap
  (fun x -> x - 2)
  (first (fun x -> x > 3) [1;3;4;2;5]);;
- : int option = Some 2
```
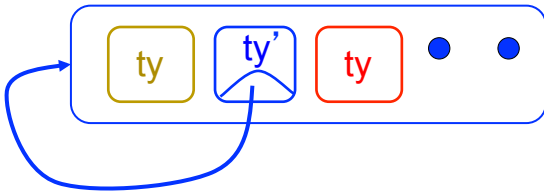
Folding over Variants

```
# let optionFold someFun noneVal opt =
    match opt with None -> noneVal
    | Some x -> someFun x;;
val optionFold : ('a -> 'b) -> 'b -> 'a option ->
  'b = <fun>
# let optionMap f opt =
    optionFold (fun x -> Some (f x)) None opt;;
val optionMap : ('a -> 'b) -> 'a option -> 'b
  option = <fun>
```

Recursive Types

- The type being defined may be a component of itself

Recursive Data Types

type exp =
    VarExp of string
  | ConstExp of const
  | MonOpAppExp of mon_op * exp
  | BinOpAppExp of bin_op * exp * exp
  | IfExp of exp* exp * exp
  | AppExp of exp * exp
  | FunExp of string * exp

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int |
…
type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp | …

- How to represent 6 as an exp?

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int |
…
type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp | …

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int |
…
type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp | …

- How to represent (6, 3) as an exp?

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int |
…
type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp | …

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp, ConstExp (IntConst 6),
                ConstExp (IntConst 3))

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
        | EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int |
…
type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp | …
- How to represent [(6, 3)] as an exp?
- BinOpAppExp (ConsOp, BinOpAppExp (CommaOp,
  ConstExp (IntConst 6), ConstExp (IntConst 3)),
  ConstExp NilConst))));;

Your turn now

Try Problem 1 on MP3

Recursive Data Types

type int_Bin_Tree =
 Leaf of int | Node of (int_Bin_Tree *
   int_Bin_Tree);;

type int_Bin_Tree = Leaf of int | Node of
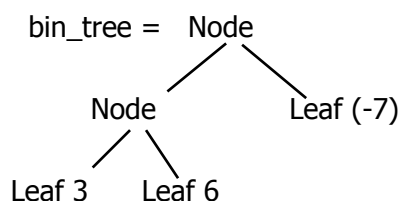   (int_Bin_Tree * int_Bin_Tree)

Recursive Data Type Values

let bin_tree =
 Node(Node(Leaf 3, Leaf 6),Leaf (-7));;

val bin_tree : int_Bin_Tree = Node (Node
   (Leaf 3, Leaf 6), Leaf (-7))

Recursive Data Type Values

bin_tree =   Node
         Node   Leaf (-7)
    Leaf 3   Leaf 6

Recursive Functions

let rec first_leaf_value tree =
    match tree with (Leaf n) -> n
    | Node (left_tree, right_tree) ->
     first_leaf_value left_tree;;
val first_leaf_value : int_Bin_Tree -> int =
   <fun>
let left = first_leaf_value bin_tree;;
val left : int = 3

Problem

type int_Bin_Tree =Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;
- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree
let rec sum_tree t =

Problem

type int_Bin_Tree =Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;
- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree
let rec sum_tree t =
     match t with Leaf n -> n
       | Node(t1,t2) -> sum_tree t1 + sum_tree t2

Mapping over Recursive Types

```
# let rec ibtreeMap f tree =
    match tree with (Leaf n) -> Leaf (f n)
    | Node (left_tree, right_tree) ->
     Node (ibtreeMap f left_tree,
         ibtreeMap f right_tree);;
val ibtreeMap : (int -> int) -> int_Bin_Tree ->
  int_Bin_Tree = <fun>
```

Mapping over Recursive Types

```
# ibtreeMap ((+) 2) bin_tree;;

- : int_Bin_Tree = Node (Node (Leaf 5, Leaf
  8), Leaf (-5))
```

Folding over Recursive Types

```
# let rec ibtreeFoldRight leafFun nodeFun tree =
    match tree with Leaf n -> leafFun n
    | Node (left_tree, right_tree) ->
     nodeFun
     (ibtreeFoldRight leafFun nodeFun left_tree)
     (ibtreeFoldRight leafFun nodeFun right_tree);;
val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) ->
  int_Bin_Tree -> 'a = <fun>
```

Folding over Recursive Types

```
# let tree_sum =
    ibtreeFoldRight (fun x -> x) (+);;
val tree_sum : int_Bin_Tree -> int = <fun>
# tree_sum bin_tree;;
- : int = 2
```