Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Grammars

- Grammars are formal descriptions of which strings over a given character set are in a particular language
- Language designers write grammar
- Language implementers use grammar to know what programs to accept
- Language users use grammar to know how to write legitimate programs

Types of Formal Language Descriptions

- Regular expressions, regular grammars
- Context-free grammars, BNF grammars, syntax diagrams
- Finite state automata
- Whole family more of grammars and automata – covered in automata theory

Sample Grammar

- Language: Parenthesized sums of 0's and 1's
- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)

BNF Grammars

- Start with a set of characters, **a,b,c,…**
  - We call these *terminals*
- Add a set of different characters, **X,Y,Z, …**
  - We call these *nonterminals*
- One special nonterminal **S** called *start symbol*

BNF Grammars

- BNF rules (aka *productions*) have form

  X ::= *y*

  where **X** is any nonterminal and *y* is a string of terminals and nonterminals
- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule

Sample Grammar

- Terminals: 0 1 + ( )
- Nonterminals: <Sum>
- Start symbol = <Sum>

- <Sum> ::= 0
- <Sum >::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= (<Sum>)
- Can be abbreviated as

<Sum> ::= 0 | 1
| <Sum> + <Sum> | (<Sum>)

BNF Deriviations

- Given rules

X::= *y***Z***w* and **Z**::=*v*

we may replace **Z** by *v* to say

X => *y***Z***w* => *yvw*

- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal

BNF Derivations

- Start with the start symbol:

<Sum> =>

BNF Derivations

- Pick a non-terminal

<Sum> =>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= (<Sum>)

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 1

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

- Pick a rule and substitute
  - <Sum> ::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) 0
=> (0 + 1) + 0

BNF Derivations

- (0 + 1) + 0 is generated by grammar

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0
=> (0 + 1) + 0

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

<Sum> =>

BNF Semantics

- The meaning of a BNF grammar is the set of all strings consisting only of terminals that can be derived from the Start symbol

Extended BNF Grammars

- Alternatives: allow rules of from X::=*y*|*z*
  - Abbreviates X::= *y*, X::= *z*
- Options: X::=*y*[*v*]*z*
  - Abbreviates X::=*yvz*, X::=*yz*
- Repetition: X::=*y*{*v*}**z*
  - Can be eliminated by adding new nonterminal V and rules X::=*yz*, X::=yVz, V::=*v*, V::=*v*V

Regular Grammars

- Subclass of BNF
- Only rules of form
  <nonterminal>::=<terminal><nonterminal> or
  <nonterminal>::=<terminal> or
  <nonterminal>::= ε
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)

Example

- Regular grammar:
  <Balanced> ::= ε
  <Balanced> ::= 0<OneAndMore>
  <Balanced> ::= 1<ZeroAndMore>
  <OneAndMore> ::= 1<Balanced>
  <ZeroAndMore> ::= 0<Balanced>
- Generates even length strings where every initial substring of even length has same number of 0's as 1's

Parse Trees

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it

Example

- Consider grammar:
  <exp> ::= <factor>
  | <factor> + <factor>
  <factor> ::= <bin>
  | <bin> * <exp>
  <bin> ::= 0 | 1
- Problem: Build parse tree for 1 * 1 + 0 as an <exp>

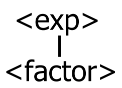
Example cont.

- 1 * 1 + 0: <exp>

<exp> is the start symbol for this parse tree

Example cont.

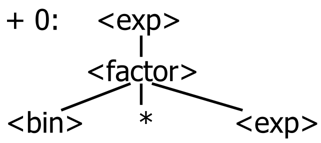
- 1 * 1 + 0:



Use rule: <exp> ::= <factor>

Example cont.

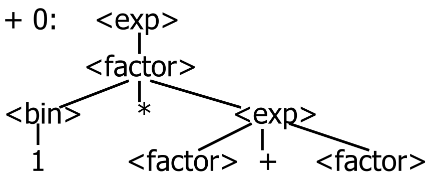
- 1 * 1 + 0:



Use rule: <factor> ::= <bin> * <exp>

Example cont.

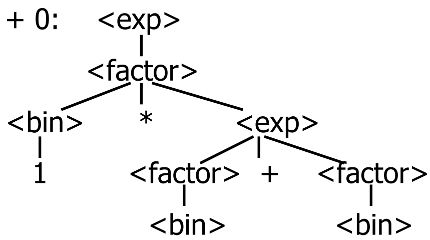
- 1 * 1 + 0:



Use rules: <bin> ::= 1 and
<exp> ::= <factor> + <factor>

Example cont.

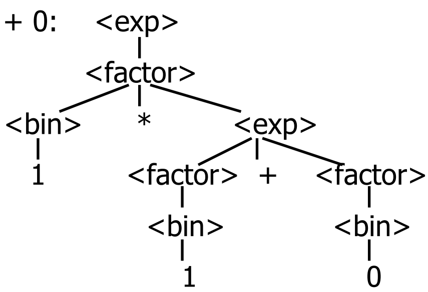
- 1 * 1 + 0:



Use rule: <factor> ::= <bin>

Example cont.

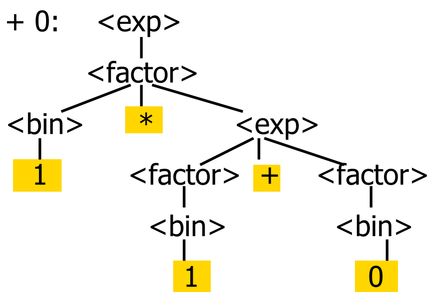
- 1 * 1 + 0:



Use rules: <bin> ::= 1 | 0

Example cont.

- 1 * 1 + 0:



Fringe of tree is string generated by grammar

Your Turn: 1 * 0 + 0 * 1

Parse Tree Data Structures

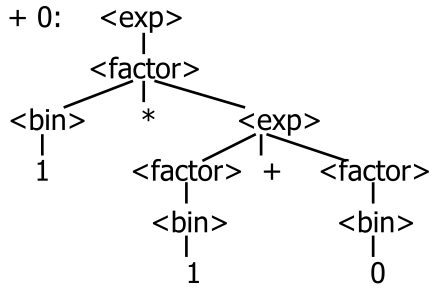
- Parse trees may be represented by OCaml datatypes
- One datatype for each nonterminal
- One constructor for each rule
- Defined as mutually recursive collection of datatype declarations

Example

- Recall grammar:
  <exp> ::= <factor> | <factor> + <factor>
  <factor> ::= <bin> | <bin> * <exp>
  <bin> ::= 0 | 1
- type exp = Factor2Exp of factor
  | Plus of factor * factor
  and factor = Bin2Factor of bin
  | Mult of bin * exp
  and bin = Zero | One

Example cont.

- 1 * 1 + 0:



Example cont.

- Can be represented as

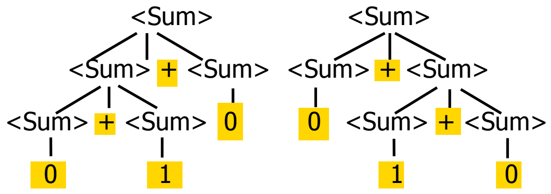
Factor2Exp
(Mult(One,
  Plus(Bin2Factor One,
    Bin2Factor Zero)))

Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree
- If all BNF's for a language are ambiguous then the language is *inherently ambiguous*

Example: Ambiguous Grammar

- 0 + 1 + 0



Example

- What is the result for:

  3 + 4 * 5 + 6

Example

- What is the result for:

  3 + 4 * 5 + 6

- Possible answers:
  - 41 = ((3 + 4) * 5) + 6
  - 47 = 3 + (4 * (5 + 6))
  - 29 = (3 + (4 * 5)) + 6 = 3 + ((4 * 5) + 6)
  - 77 = (3 + 4) * (5 + 6)

Example

- What is the value of:

  7 – 5 – 2

Example

- What is the value of:

  7 – 5 – 2

- Possible answers:
  - In Pascal, C++, SML assoc. left

    7 – 5 – 2 = (7 – 5) – 2 = 0
  - In APL, associate to right

    7 – 5 – 2 = 7 – (5 – 2) = 4

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoicativity

- Not the only sources of ambiguity

How to Enforce Associativity

- Have at most one recursive call per production
- When two or more recursive calls would be natural leave right-most one for right assoicativity, left-most one for left assoiciativity

Example

- <Sum> ::= 0 | 1 | <Sum> + <Sum>
  | (<Sum>)
- Becomes
  - <Sum> ::= <Num> | <Num> + <Sum>
  - <Num> ::= 0 | 1 | (<Sum>)

Operator Precedence

- Operators of highest precedence evaluated first (bind more tightly).
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

Precedence Table - Sample

	Fortan	Pascal	C/C++	Ada	SML
highest	**	*, /, div, mod	++, --	**	div, mod, /, *
	*, /	+, -	*, /, %	*, /, mod	+, -, ^
	+, -		+, -	+, -	::

First Example Again

- In any above language, 3 + 4 * 5 + 6 = 29
- In APL, all infix operators have same precedence
  - Thus we still don't know what the value is (handled by associativity)
- How do we handle precedence in grammar?

Predence in Grammar

- Higher precedence translates to longer derivation chain
- Example:

<exp> ::= <id> | <exp> + <exp>
| <exp> * <exp>

- Becomes

<exp> ::= <mult_exp>
| <exp> + <mult_exp>
<mult_exp> ::= <id> | <mult_exp> * <id>