Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Recall

```
# let rec poor_rev list = match list
  with [] -> []
    | (x::xs) -> poor_rev xs @ [x];;
val poor_rev : 'a list -> 'a list = <fun>
```

Tail Recursion - Example

```
# let rec rev_aux list revlist =
  match list with [ ] -> revlist
  | x :: xs -> rev_aux xs (x::revlist);;
val rev_aux : 'a list -> 'a list -> 'a list = <fun>


# let rev list = rev_aux list [ ];;
val rev : 'a list -> 'a list = <fun>
```

Comparison

- poor_rev [1,2,3] =
- (poor_rev [2,3]) @ [1] =
- ((poor_rev [3]) @ [2]) @ [1] =
- (((poor_rev [ ]) @ [3]) @ [2]) @ [1] =
- ((([ ] @ [3]) @ [2]) @ [1]) =
- ([3] @ [2]) @ [1] =
- (3:: ([ ] @ [2])) @ [1] =
- [3,2] @ [1] =
- 3 :: ([2] @ [1]) =
- 3 :: (2:: ([ ] @ [1])) = [3, 2, 1]

Comparison

- rev [1,2,3] =
- rev_aux [1,2,3] [ ] =
- rev_aux [2,3] [1] =
- rev_aux [3] [2,1] =
- rev_aux [ ] [3,2,1] = [3,2,1]

Folding

let rec fold_left f a list = match list
  with [] -> a | (x :: xs) -> fold_left f (f a x) xs;;
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a =
    <fun>

$$fold\_left\ f\ a\ [x_1;\ x_2;...;x_n] = f(...(f\ (f\ a\ x_1)\ x_2)...)x_n$$

let rec fold_right f list b = match list
  with [ ] -> b | (x :: xs) -> f x (fold_right f xs b);;
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b =
    <fun>

$$fold\_right\ f\ [x_1;\ x_2;...;x_n]\ b = f\ x_1(f\ x_2\ (...(f\ x_n\ b)...))$$

Folding - Tail Recursion

- # let rev list =
-      fold_left
-        (fun x -> fun l -> x :: l)   //comb op
-        []   //accumulator cell
-        list

Folding

- Can replace recursion by fold_right in any forward primitive recursive definition
  - Primitive recursive means it only recurses on immediate subcomponents of recursive data structure
- Can replace recursion by fold_left in any tail primitive recursive definition

Map from Fold

let map f list =
 fold_right (fun x y -> f x :: y) list [ ];;
val map : ('a -> 'b) -> 'a list -> 'b list =
   <fun>
map ((+)1) [1;2;3];;
- : int list = [2; 3; 4]

- Can you write fold_right (or fold_left) with just map? How, or why not?

Higher Order Functions

- A function is *higher-order* if it takes a function as an argument or returns one as a result
- Example:

let compose f g = fun x -> f (g x);;

val compose : ('a -> 'b) -> ('c -> 'a) -> 'c -> 'b = <fun>

- The type ('a -> 'b) -> ('c -> 'a) -> 'c -> 'b is a higher order type because of ('a -> 'b) and ('c -> 'a) and -> 'c -> 'b

Partial Application

(+);;

- : int -> int -> int = <fun>

(+) 2 3;;

- : int = 5

let plus_two = (+) 2;;

val plus_two : int -> int = <fun>

plus_two 7;;

- : int = 9

- Patial application also called *sectioning*

Lambda Lifting

- You must remember the rules for evaluation when you use partial application

let add_two = (+) (print_string "test\n"; 2);;

test

val add_two : int -> int = <fun>

let add2 =   (* lambda lifted *)

   fun x -> (+) (print_string "test\n"; 2) x;;

val add2 : int -> int = <fun>

Lambda Lifting

\# thrice add_two 5;;

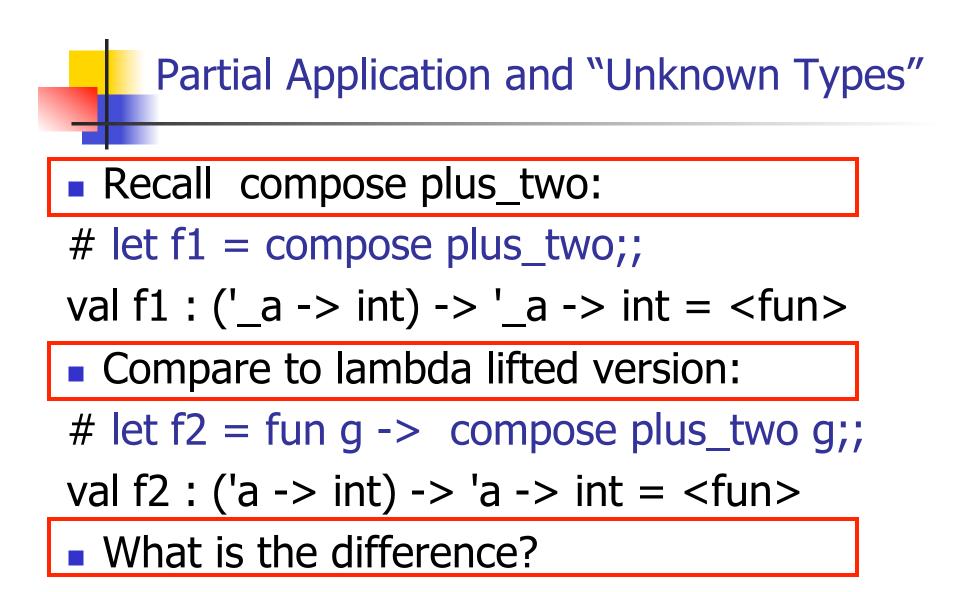- : int = 11

\# thrice add2 5;;

test

test

test

- : int = 11

- Lambda lifting delayed the evaluation of the argument to (+) until the second argument was supplied

Partial Application and "Unknown Types"

- Recall compose plus_two:

let f1 = compose plus_two;;

val f1 : ('_a -> int) -> '_a -> int = <fun>

- Compare to lambda lifted version:

let f2 = fun g -> compose plus_two g;;

val f2 : ('a -> int) -> 'a -> int = <fun>

- What is the difference?

Partial Application and "Unknown Types"

- `_a can only be instantiated once for an expression

\# f1 plus_two;;

- : int -> int = <fun>

\# f1 List.length;;

Characters 3-14:

  f1 List.length;;

     ^^^^^^^^^^^

This expression has type 'a list -> int but is here used with type int -> int

Partial Application and "Unknown Types"

- `a can be repeatedly instantiated

f2 plus_two;;
- : int -> int = <fun>
f2 List.length;;
- : '_a list -> int = <fun>

Continuations

- Idea: Use functions to represent the control flow of a program

- Method: Each procedure takes a function as an argument to which to pass its result; outer procedure "returns" no result

- Function receiving the result called a continuation

- Continuation acts as "accumulator" for work still to be done

Example of Tail Recursion

```
# let rec prod l =
     match l with [] -> 1
     | (x :: rem) -> x * prod rem;;
val prod : int list -> int = <fun>
# let prod list =
     let rec prod_aux l acc =
          match l with [] -> acc
          | (y :: rest) -> prod_aux rest (acc * y)
(* Uses associativity of multiplication *)
     in prod_aux list 1;;
 val prod : int list -> int = <fun>
```

Example of Tail Recursion

```
# let rec app fl x =
    match fl with [] -> x
    | (f :: rem_fs) -> f (app rem_fs x);;
val app : ('a -> 'a) list -> 'a -> 'a = <fun>
# let app fs x =
    let rec app_aux fl acc=
        match fl with [] -> acc
        | (f :: rem_fs) -> app_aux rem_fs
                    (fun z -> acc (f z))
    in app_aux fs (fun y -> y) x;;
val app : ('a -> 'a) list -> 'a -> 'a = <fun>
```

Continuation Passing Style

- Writing procedures so that they take a continuation to which to give (pass) the result, and return no result, is called continuation passing style (CPS)

Example of Tail Recursion & CSP

```
# let app fs x =
    let rec app_aux fl acc=
        match fl with [] -> acc
        | (f :: rem_fs) -> app_aux rem_fs
                            (fun z -> acc (f z))
    in app_aux fs (fun y -> y) x;;
val app : ('a -> 'a) list -> 'a -> 'a = <fun>
# let rec appk fl x k =
    match fl with [] -> k x
    | (f :: rem_fs) -> appk rem_fs x (fun z -> k (f z));;
val appk : ('a -> 'a) list -> 'a -> ('a -> 'b) -> 'b
```

Example of CSP

```
# let rec app fl x =
    match fl with [] -> x
    | (f :: rem_fs) -> f (app rem_fs x);;
val app : ('a -> 'a) list -> 'a -> 'a = <fun>

# let rec appk fl x k =
    match fl with [] -> k x
    | (f :: rem_fs) -> appk rem_fs x (fun z -> k (f z));;
val appk : ('a -> 'a) list -> 'a -> ('a -> 'b) -> 'b =
    <fun>
```

Continuation Passing Style

- A programming technique for all forms of "non-local" control flow:
    - non-local jumps
    - exceptions
    - general conversion of non-tail calls to tail calls
- Essentially it's a higher-order function version of GOTO

Continuation Passing Style

- A compilation technique to implement non-local control flow, especially useful in interpreters.

- A formalization of non-local control flow in denotational semantics

Terms

- A function is in Direct Style when it returns its result back to the caller.

- A Tail Call occurs when a function returns the result of another function call without any more computations (eg tail recursion)

- A function is in Continuation Passing Style when it passes its result to another function.

- Instead of returning the result to the caller, we pass it forward to another function.

Example

- Simple reporting continuation:

```
# let report x = (print_int x; print_newline( ) );;
val report : int -> unit = <fun>
```

- Simple function using a continuation:

```
# let plusk a b k = k (a + b)
val plusk : int -> int -> (int -> 'a) -> 'a = <fun>
# plusk 20 22 report;;
42
- : unit = ()
```