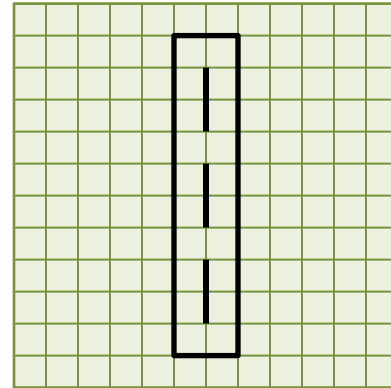Name: _____   Due: 3:30pm, 28 Feb. 2012

ID: _____

Rules: *Your work and answers must be your own. For maximum credit, show all work.*

1. [60] Consider a 2-D model of a landing strip in the xy-plane (z=0), consisting of the rectangle with 3-D vertices (-1,0,0),(1,0,0),(1,10,0),(-1,10,0) and line segments extending from (0,1,0) to (0,3,0), from (0,4,0) to (0,6,0), and from (0,7,0) to (0,9,0).

    a. [5] Construct a 4x4 homogeneous modeling transformation to transform this model into a world coordinate system where the xz plane is the ground and the y-axis extends upwards. This modeling transformation should embed the 2-D model in the xz plane such that the center of the landing strip is at the world coordinate origin, and the striped line extends along the z axis. This transformation should map the model coordinates vertex (1,10,0) to the world coordinates vertex (1,0,5). Transform the rest of the landing strip vertices from their model coordinate positions into their world coordinates position.

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & -5 \\ & & & 1 \end{bmatrix}\begin{bmatrix} 1 & & & \\ & 0 & -1 & \\ & 1 & 0 & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 0 & -1 & \\ & 1 & 0 & -5 \\ & & & 1 \end{bmatrix}$$

or $\begin{bmatrix} 1 & & & \\ & 0 & -1 & \\ & 1 & 0 & \\ & & & 1 \end{bmatrix}\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & -5 \\ & & & 1 \end{bmatrix}$

(-1,0,-5),(1,0,-5),(1,0,5),(-1,0,5),(0,0,-4),(0,0,-2),(0,0,-1),(0,0,1),(0,0,2),(0,0,4)

    b. [5] If the eye is positioned at the point (-.5,1,6) in world coordinates looking at the origin, then construct *T*, the 4x4 homogeneous transformation matrix that translates the eye position in world coordinates to the origin of the viewing coordinate system.

$$T = \begin{bmatrix} 1 & & & .5 \\ & 1 & & -1 \\ & & 1 & -6 \\ & & & 1 \end{bmatrix}$$

c. [5] Let (0,1,0) be the "up" direction in world coordinates. Calculate *unit length* vectors in world coordinates corresponding to

   i. the view vector **v** from the eye toward the lookat point

   v = normalize(lookat – eye = (0,0,0) – (-.5,1,6) = (.5,-1,-6)) = (.0819,-.164,-.983)

   ii. the vector **r** perpendicular to the plane containing **v** and **up**

   r = (v x up = (.983,0,.0819))/|v x up| = (.997,0,.083)

   iii. the vector **u** perpendicular to **v** and **r**.

   u = r x v = (.0136,.986,-.163)

   Verify that <v,u,r> form a right-handed perpendicular coordinate system. (no points, but good way to check your work.)

   v.r = r.u = r.v = 0. v x u = r, u x r = v, r x v = u.

d. [5] Construct a 4x4 homogeneous transformation matrix *R* that rotates vector **r** into the +*x* direction, vector **u** into the +*y* direction and vector **v** into the –*z* direction, and verify that it does so by computing the products $R\mathbf{r} = (1,0,0,0)^T$, $R\mathbf{u} = (0,1,0,0)^T$ and $R\mathbf{v} = (0,0,-1,0)^T$.

$$T = \begin{bmatrix} .997 & 0 & .083 & \\ .0136 & .986 & -.163 & \\ -.0819 & .163 & .983 & \\ & & & 1 \end{bmatrix}$$

e. [5] Construct a 4x4 homogeneous viewing transformation *V* using *R* and *T*.

$$\begin{bmatrix} .997 & 0 & .083 & \\ .0136 & .986 & -.163 & \\ -.0819 & .163 & .983 & \\ & & & 1 \end{bmatrix}\begin{bmatrix} 1 & & & .5 \\ & 1 & & -1 \\ & & 1 & -6 \\ & & & 1 \end{bmatrix} = \begin{bmatrix} .997 & 0 & .083 & \\ .0136 & .986 & -.163 & \\ -.0819 & .163 & .983 & -6.1 \\ & & & 1 \end{bmatrix}$$

f. [5] Use V to transform the world coordinate vertices to viewing coordinate vertices. The world coordinates of the third vertex (1,0,5) should map to the viewing coordinate position (1.41, -0.803, -1.27).

{-1.41177, 0.802795, -10.9367, 1.},
{0.581318, 0.830009, -11.1006, 1.},
{1.41177, -0.802795, -1.26981, 1.},
{-0.581318, -0.830009, -1.10596, 1.},
{-0.332182, 0.653122, -10.0356, 1.},
{-0.166091, 0.326561, -8.06943, 1.},
{-0.0830455, 0.16328, -7.08636, 1.},
{0.0830455, -0.16328, -5.1202, 1.},
{0.166091, -0.326561, -4.13712, 1.}
{0.332182, -0.653122, -2.17096, 1.}

g. [5] Plot the x and y coordinates of the viewing coordinate vertices on graph paper to show an orthographics view of the runway from the given view.
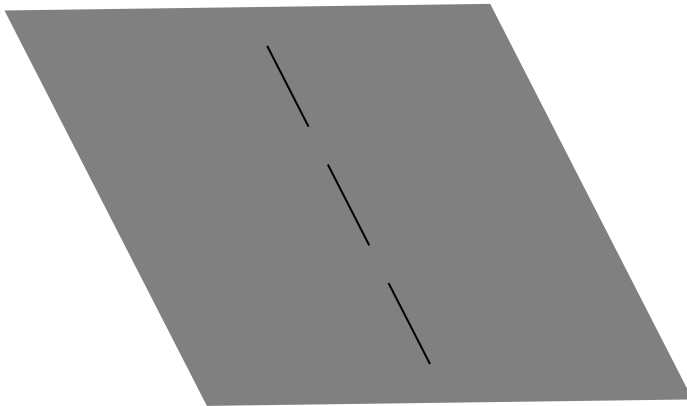

h. [5] Construct a perspective projection matrix where the distance of the canvas (window) from the eye is 1.25 units (d = 1.25).

1

   1

      1

      -0.8   0

i. [5] Apply the perspective projection matrix to the viewing coordinates and show the resulting homogeneous 4-vectors for each vertex, and for each vertex indicate what (x,y) coordinate it corresponds to in the z = -0.8 projection plane by performing a homogeneous divide (so the resulting homogeneous coordinate equals 1). For example, the homogeneous viewing coordinates of the third vertex, $(1.41, -0.803, -1.27, 1)^T$, will be transformed by the perspective projection to $(1.41, -0.803, -1.27, 1.02)^T$ in homogeneous "clip" coordinates, which corresponds to the point (1.41/1.02, -0.803/1.02) = (1.39, -0.790) in canvas coordinates.

{-1.41177, 0.802795, -10.9367, 8.7494} → {-0.161357, 0.0917544},

{0.581318, 0.830009, -11.1006, 8.88047} → {0.0654603, 0.0934645},

{1.41177, -0.802795, -1.26981, 1.01585} → {1.38975, -0.790272},

{-0.581318, -0.830009, -1.10596, 0.88477} → {-0.657027, -0.938106},

{-0.332182, 0.653122, -10.0356, 8.02847} → {-0.0413755, 0.0813507},

{-0.166091, 0.326561, -8.06943, 6.45555} → {-0.0257284, 0.0505861},

{-0.0830455, 0.16328, -7.08636, 5.66908} → {-0.0146488, 0.0288019},

{0.0830455, -0.16328, -5.1202, 4.09616} → {0.020274, -0.0398618},

{0.166091, -0.326561, -4.13712, 3.3097} → {0.0501831, -0.0986679},

{0.332182, -0.653122, -2.17096, 1.73677} → {0.191264, -0.376055}

j. [5] Plot the resulting perspective projection of the landing strip on graph paper.
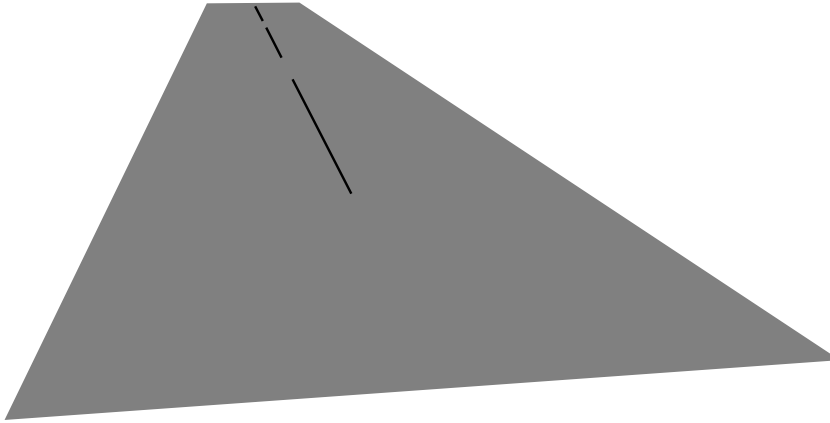
k. [10] The canvas coordinates of the third vertex (1.39, -0.790) is outside the canvas clipping region (-1,-1),(1,-1),(1,1),(-1,1). Clip the segments that have this third vertex as an endpoint to the right clipping edge x = 1 and compute the new vertices and segments needed for the clipped landing strip. Plot these new vertices and segments on the previous perspective plot.

{0.0654603, 0.0934645}, {1.38975, -0.790272}, {-0.657027, -0.938106}

x(t) = 0.0655 + t(1.39 - 0.0655) = 0.0655 + 1.32t
y(t) = 0.0935 + t(-.790 – 0.0935) = 0.095 – 0.884t
1 = 0.0655 + 1.32t → t = (1 – 0.0655)/1.32 = .708
y(0.708) = 0.096 - 0.884(0.708) = -0.530
new vertex at (1,-0.530)

x(t) = 1.39 + t(-0.657 – 1.39) = 1.39 – 2.05t
y(t) = -.790 + t(-.938 - -.790) = -.790 - .148t
1 = 1.39 – 2.05t → t = (1 – 1.39)/(-2.05) = .190
y(.190) = -.790 - .148(.190) = -.818
new vertex at (1,-0.818)

(0.0655,0.0935),(1,-.0530),(1,-0.818),(-0.647,-0.938)

2. [20] Consider the triangle with vertices (1,0,0),(0,0,1),(1,1,1), a point light source located at the point (0,2,0) with power one, and an eye point at (3,0,0). Assume no attenuation of light in these calculations.

a. [5] What is the (unit-length) surface normal for the side of the triangle facing the light source?

(0,1,1) x (1,1,0) = (-1,1,-1)/sqrt(3) = (-.577,.577,-.577)

b. [5] According to Lambert's law and assuming a constant coefficient of diffuse reflection k_d = 1 over the triangle, which vertex diffusely reflects the most light toward the viewer? How much light is reflected toward the viewer by this vertex?

Vertex (1,1,1) diffusely reflects the most light. [5 points]
Light vector L at (1,1,1) is (0,2,0) – (1,1,1) = (-1,1,-1) unitized as (-.577,.577,-.577), so this vertex reflects one unit of light (all the light) toward the viewer. [5 points]

c. [5] What is the (unit-length) reflection direction measured at the vertex (1,1,1)? Why?

R = L + 2((N.L)N – L) = 2(N.L)N – L
R = 2(-.577,.577,-.577).(-.577,.577,-.577) (-.577,.577,-.577) – (-.577,.577,-.577)
  = (-.577,.577,-.577)
Since N = L, the reflection vector R also equals L.

d. [5] Assume a constant specular reflection coefficient of .3, and specular shininess power n = 10, then how much light is reflected from the (Phong) specular gleam at vertex (0,0,1) toward the viewer?

L = (0,2,-1) = (0,.894,-.447) at vertex (1,0,0).
N.L = (-.577,.577,-.577).(0,.894,-.447) = .774
S = .774*(-.577,.577,-.577) – (0,.894,-.447) = (-.447,-.447,0)
R = L+2S = (0,.894,-.447) + (-.894,-.894,0) = (-.894,0,-.447) [5 points]
V = (3,0,-1) unitized = (.949,0,-.316)
V.R = (.949,0,-.316).(-.894,0,-.447) = -.707
0% (no specular reflection) [5 points]

3. [20] Consider a squash transformation that scales only the y component by half, and leaves the x and z components alone.

a. [5] What is the 4x4 homogeneous transformation matrix that implements this squash transformation?

<div style="color:red">

1
   .5
      1
        1

</div>

b. [5] What are the locations of the three vertices (1,0,0),(0,0,1),(1,1,1) after the squash transformation?

(1,0,0),(0,0,1),(1,.5,1)

c. [5] What would the surface normal of this new, squashed triangle be, computed using the new vertex locations?

((0,0,1) – (1,0,0)) x ((1,.5,1) – (1,0,0))
= (-1,0,1) x (0,.5,1) = (-.5,1,-.5)
Unitized gives
N = (-.408,.816,-.408)
the vector (.408,-.816,.408) is also acceptable

d. [5] Compute this same new surface normal by computing the inverse transpose (or the adjoint transpose) of the squash transformation matrix, and apply it to the original surface normal to find the new surface normal. Don't forget to re-unitize the new surface normal.

<div style="color:red">

1
   2
      1
        1

</div>

Original normal N = (-.577,.577,-.577) so new normal N' = M^-1T*N = (-.577,1.154,-.577) normalized = (-.408,.816,-.408).