

# MP4: Smooth I

CS 418 – Interactive Computer Graphics  
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Fall 2012

# Today's Topics

- **MP4 Explanation**

- **Subdivision 40%**

- Curved camera path 10%
    - Appearance (texture/lighting/color) 10%

- **Compilation 20%**

- **Documentation 20%**

- **Non-manifold mesh**

- **catmul-clark subdivision coding scheme**

- **camera transitions**

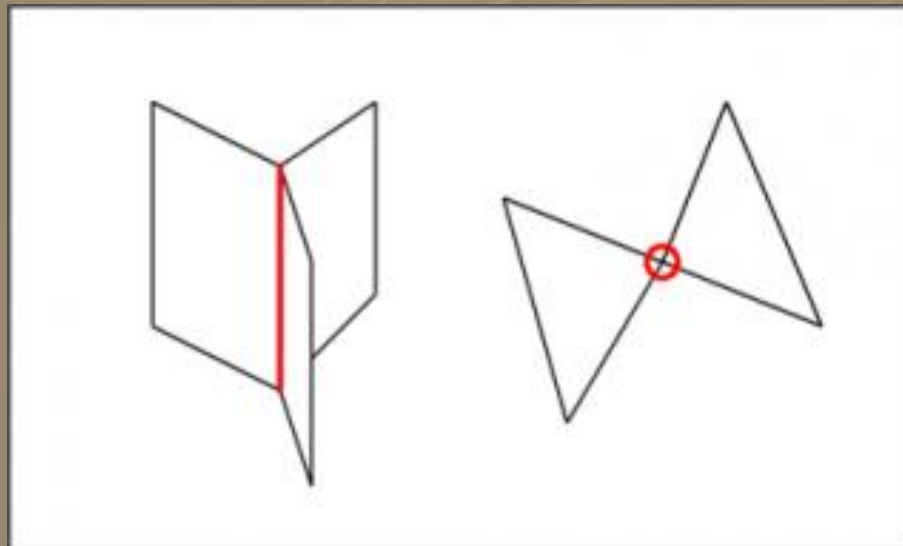
- **More about particles**

# Non-Manifold Mesh

- A "Non-Manifold" mesh is a mesh for which there are edges belonging to *more* than two faces.
- In general a "Non-Manifold" mesh occurs when you have internal faces and the like.

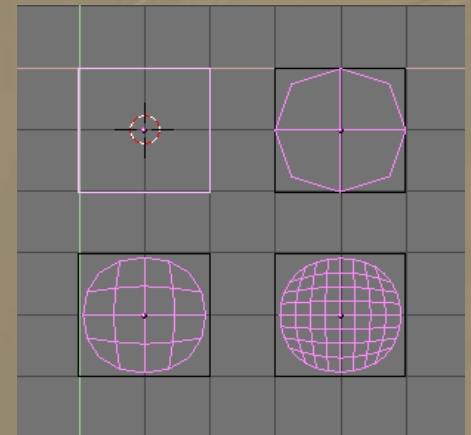
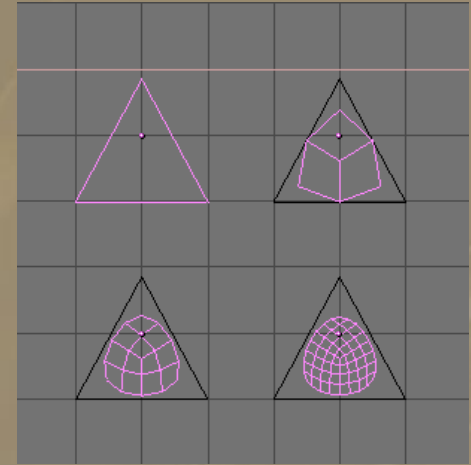
→ **(make sure you have a manifold mesh)**

- [http://www.youtube.com/watch?feature=player\\_embedded&v=vrqxp89iIM4](http://www.youtube.com/watch?feature=player_embedded&v=vrqxp89iIM4)



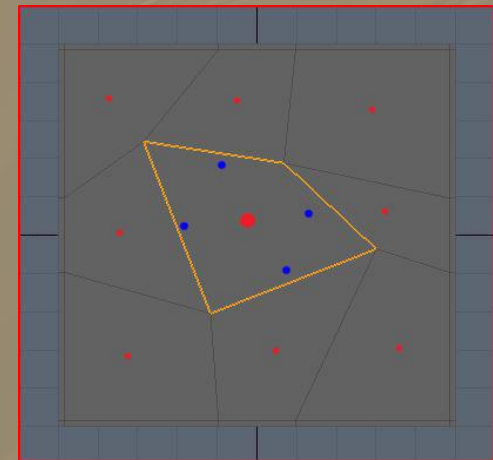
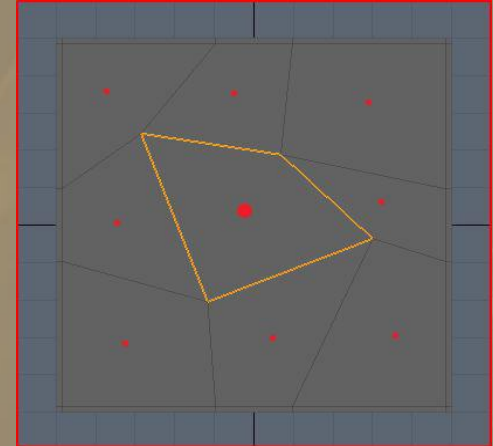
# catmul-clark subdivision

- Start with a manifold mesh.
- All the vertices in the mesh are called original points.
- Loops on:
  1. each face
  2. each edge
  3. each original point  $P$
- Connect the new points



## catmul-clark subdivision (cont.)

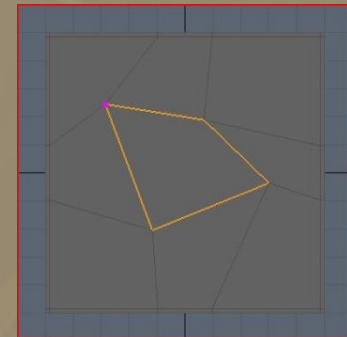
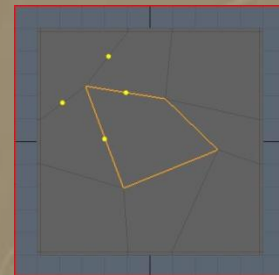
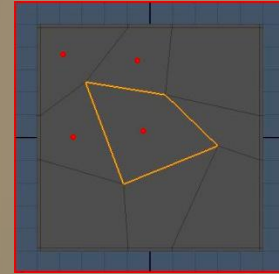
- For each face, add a *face point*
  - Set each face point to be the centroid of all original points for the respective face.
- For each edge, add an *edge point*
  - Set each edge point to be the *average* of the two neighbouring face points and its two original endpoints.



## catmul-clark subdivision (cont.)

- For each original point  $P$ :
  - $F$  = average  $F$  of all  $n$  face points for faces touching  $P$
  - $R$  = average  $R$  of all  $n$  edge midpoints for edges touching  $P$ 
    - each edge midpoint is the average of its two endpoint vertices.
  - “**Move**” each original point to the point ( $n=4$ ):

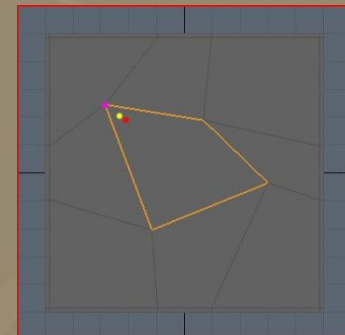
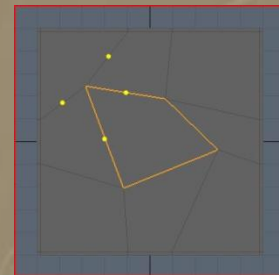
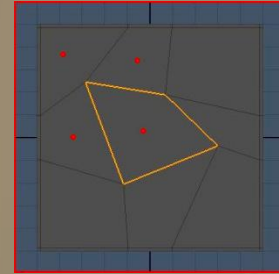
$$\frac{F + 2R + (n - 3)P}{n}$$



# catmul-clark subdivision (cont.)

- For each original point  $P$ :
  - $F$  = average  $F$  of all  $n$  face points for faces touching  $P$
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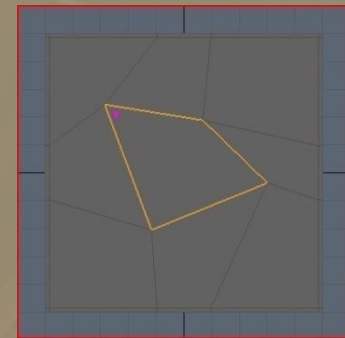
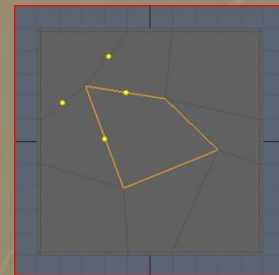
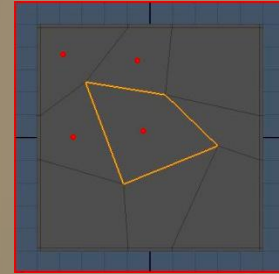
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## catmul-clark subdivision (cont.)

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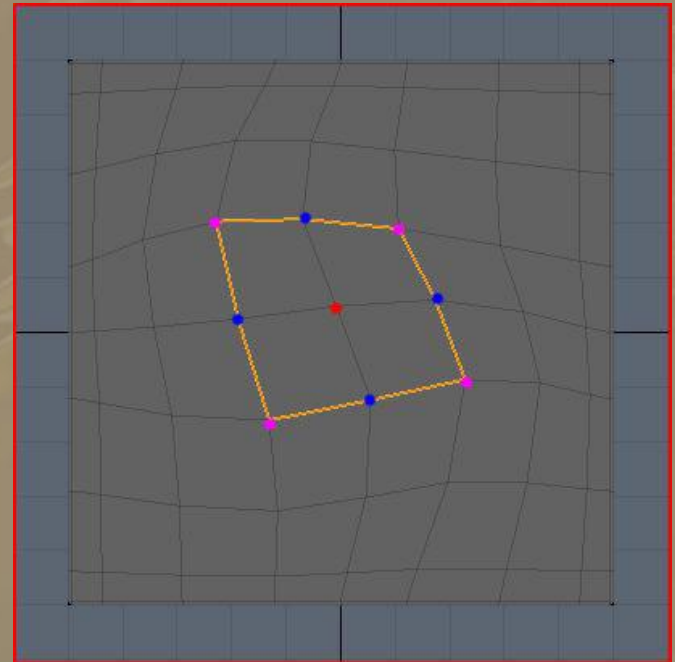
$$\frac{F + 2R + (n - 3)P}{n}$$





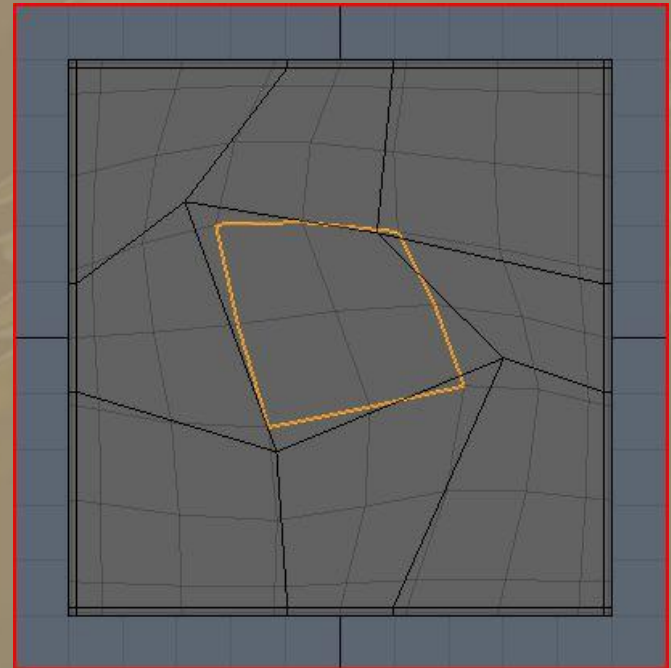
# catmul-clark subdivision (cont.)

- **Connect all points:**
  - **Blue** = (new) edge points
  - **Red** = (new) face point
  - **Pink** = (modified) vertex



# catmul-clark subdivision (cont.)

- **Connect all points:**
  - **Blue** = (new) edge points
  - **Red** = (new) face point
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# Sharp Edges

## 1. Tag Edges as “sharp” or “not-sharp”

- $sh = 0$  “not sharp”
- $sh > 0$  sharp

## 2. During Subdivision,

- if an edge is “sharp”, use sharp subdivision rules. Newly created edges, are assigned a sharpness of  $sh-1$ .
- If an edge is “not-sharp”, use normal smooth subdivision rules.

# Sharp Rules

○ FACE (unchanged)  $f = \frac{1}{n} \sum_1^n v_i$

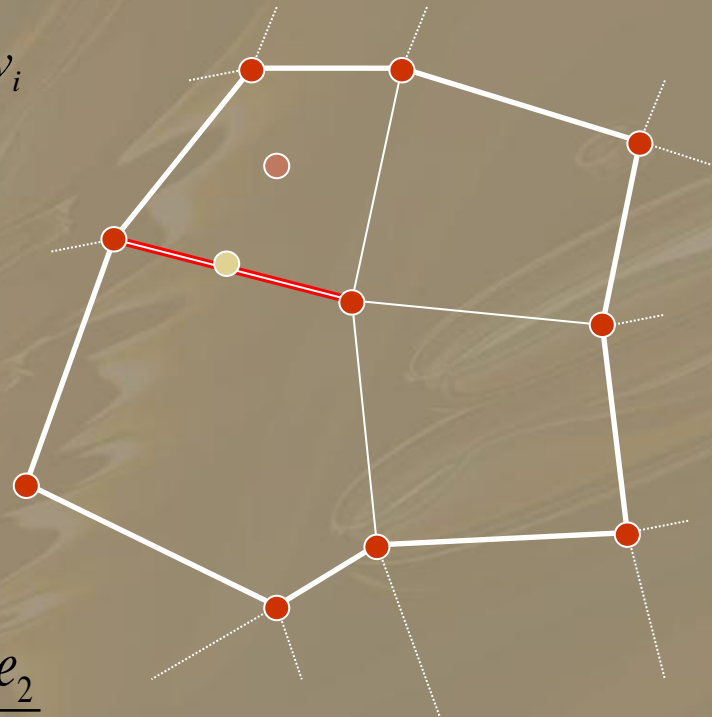
● EDGE  $e = \frac{v_1 + v_2}{2}$

● → ● VERTEX

Dart  $>2$   $v_{i+1} = v_i$   
(One sharp incident edge)

Crease  $2$   $v_{i+1} = \frac{e_1 + 6v_i + e_2}{8}$   
(Two sharp edges)

Corner  $0,1$   $v_{i+1} = \frac{n-2}{n} v_i + \frac{1}{n^2} \sum_j e_j + \frac{1}{n^2} \sum_j f_j$   
(Three or more sharp edges)

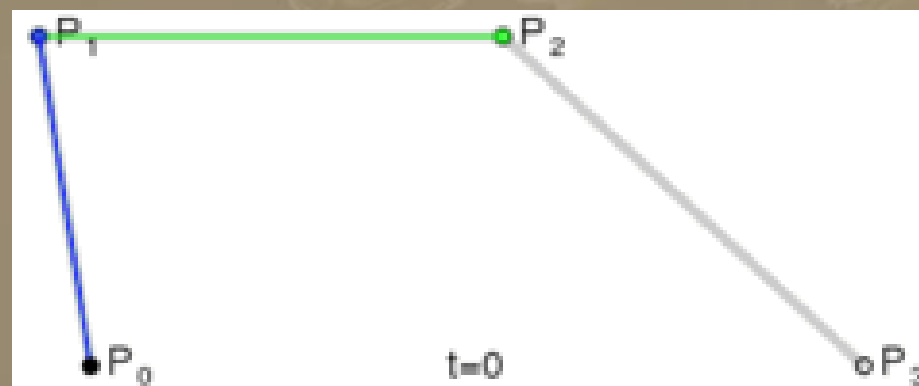
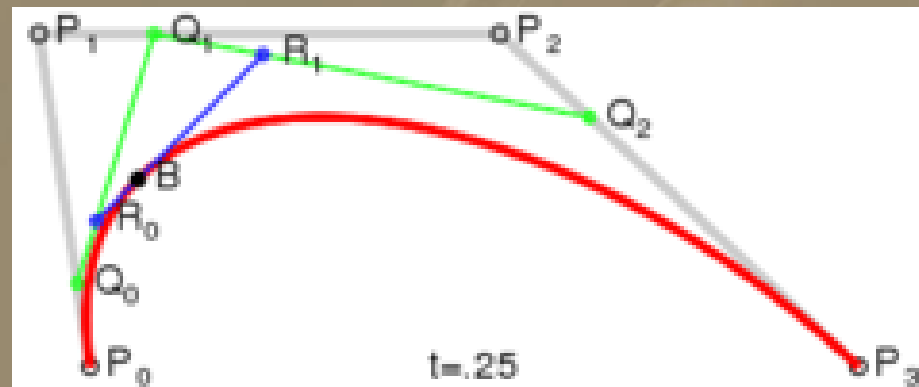


# Camera Position Update

- **Generate Random Key points:**
  - Make sure the points don't go inside the "I"
- **Interpolate Between the key points using:**
  - B-spline
  - or
  - Bezier

# Cubic Bezier Curve

$$B(t) = (1-t)^3P_0 + 3(1-t)^2tP_1 + 3(1-t)t^2P_2 + t^3P_3, t \in [0, 1]$$



Ref: "Subdivision Surfaces", Geri's Game (1989) : Pixar Animation Studios

# Continuity

$C^0$  continuity

$$p_3 = q_0$$

$C^1$  continuity

$$p_3 = q_0$$

$$p_3 - p_2 = q_1 - q_0$$

$G^1$  continuity

$$p_3 = q_0$$

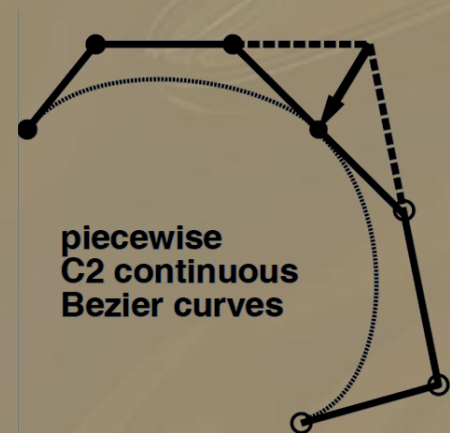
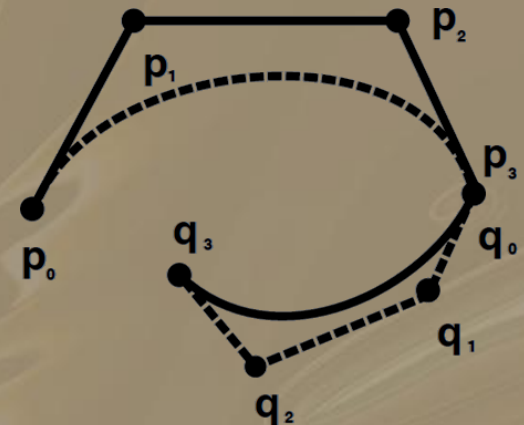
$$p_3 - p_2 = \alpha(q_1 - q_0)$$

$C^2$  continuity

$$p_3 = q_0$$

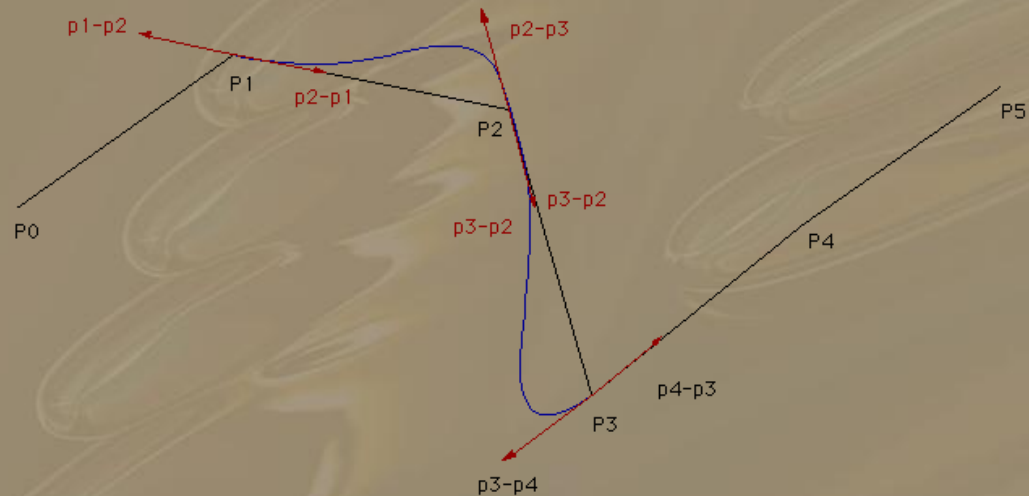
$$p_3 - p_2 = q_1 - q_0$$

$$p_3 - 2p_2 + p_1 = q_2 - 2q_1 + q_0$$



# Achieving $C^2$ Continuity

- Find tangent vectors: differences between subsequent key-frame points
  - for example: for the segment between  $p_1$  and  $p_2$  the four points use for the Bézier would be  $p_1, p_2, 2p_2-p_3, p_3$



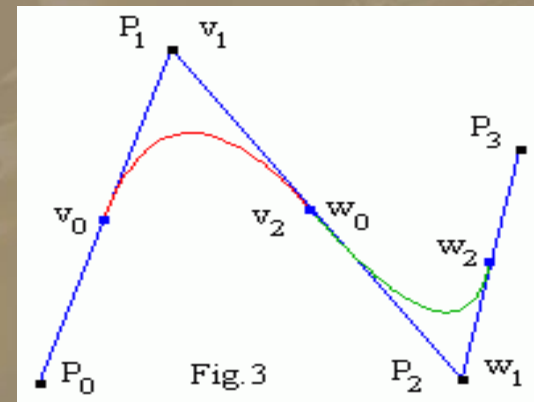
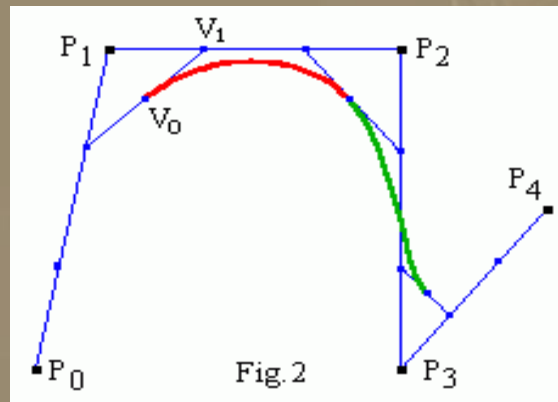
- You can also use the de-Casteljau Algorithm



# Cubic B-Spline

$$S_i(t) = \sum_{k=0}^3 P_{i-3+k} b_{i-3+k,3}(t) ; t \in [0, 1]$$

- $S_i$  is the  $i^{\text{th}}$  B-spline segment
- $P$  is the set of control points
- segment  $i$  and  $k$  is the local control point index



- Again for continuity you can use de-Boor's alg.

# Hooks Spring Law:

- Two ways:
  - Edges are considered as springs
  - If you don't want to worry about edges you can consider it's neighbor with all vertices...

$$\mathbf{f} = - \left[ k_s (\| \mathbf{x}_a - \mathbf{x}_b \| - r) + k_d (\mathbf{v}_a - \mathbf{v}_b) \frac{\mathbf{x}_a - \mathbf{x}_b}{\| \mathbf{x}_a - \mathbf{x}_b \|} \right] \frac{\mathbf{x}_a - \mathbf{x}_b}{\| \mathbf{x}_a - \mathbf{x}_b \|}$$

$k_s$  = spring constant

$k_d$  = damping constant

$r$  = rest length

# Gravitational Attraction

- **Two ways:**
  - neighbors the have an edge with it
  - all the particles: you need to calculate the average position and add all the mass and consider that as one neighboring particle

$$f = \frac{G m_a m_b}{\|x_a - x_b\|^2} \frac{x_a - x_b}{\|x_a - x_b\|}$$

$G$  = universal gravitational constant =  $6.672 \times 10^{-11} \text{N m}^2 \text{kg}^{-2}$

# Repelling Based on Charge

- particles could have a charge:

**Repel** : if the charges are the same sign

**Attract** : if they are the opposite sign

$$f = \frac{k |q_a| |q_b|}{\|x_a - x_b\|^2} \frac{x_a - x_b}{\|x_a - x_b\|}$$

$k$  = Coulombs constant =  $8.9875 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$