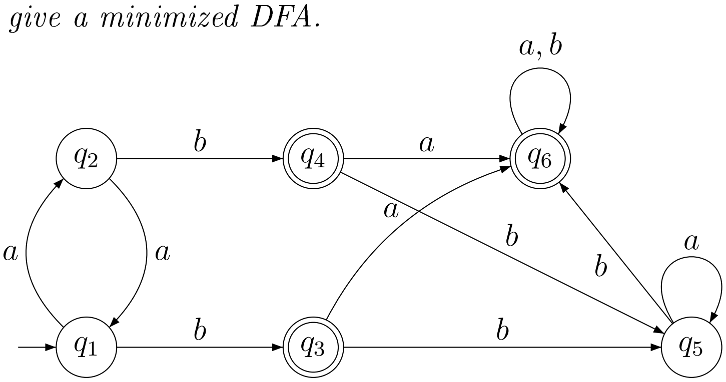
Problem Set 5

Fall 11

Due: Tuesday, 1st November, 2011, 11:00 am before class begins
Please follow the homework format guidelines posted on the class web page:
`http://www.cs.illinois.edu/class/fa11/cs373/`

1. [**Category**: DFA minimization, **Points**: 20]

Minimize the above DFA using the partition refinement technique (see lecture notes). You must show the partitions in each step and the final minimized DFA as a diagram. *Do not just give a minimized DFA.*



Solution:

Divide into two groups: non-final states and final states:

$\{\{q1, q2, q5\}, \{q3, q4, q6\}\}$

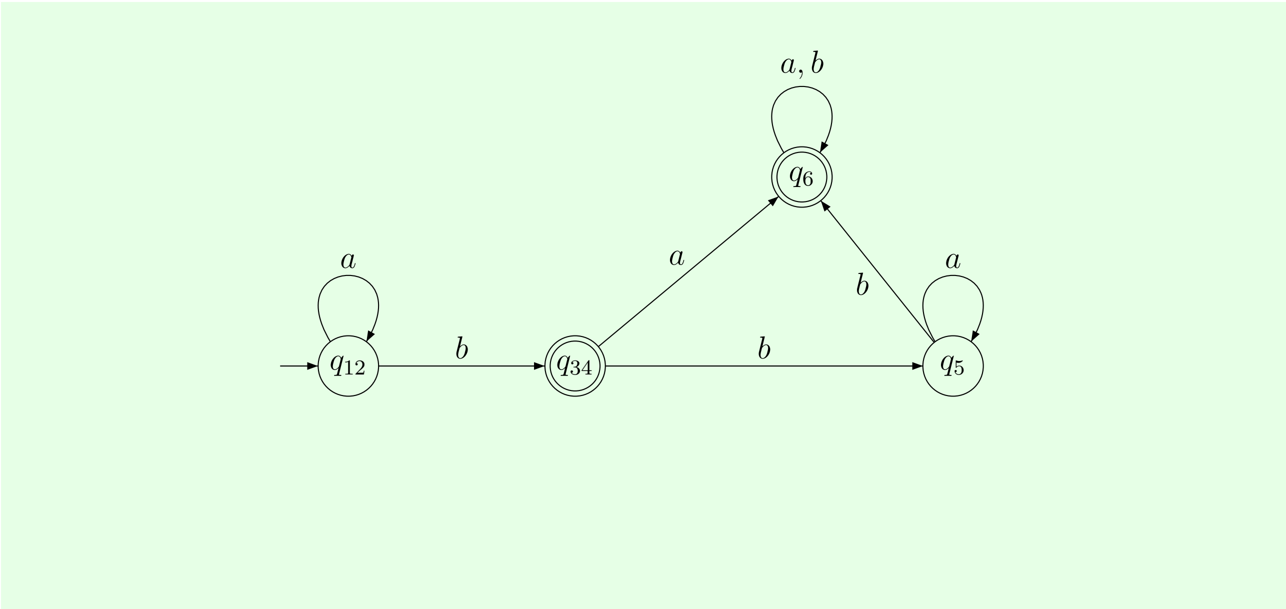
Check on input b, divide the second element into two sets:

$\{\{q1, q2, q5\}, \{q3, q4\}, \{q6\}\}$

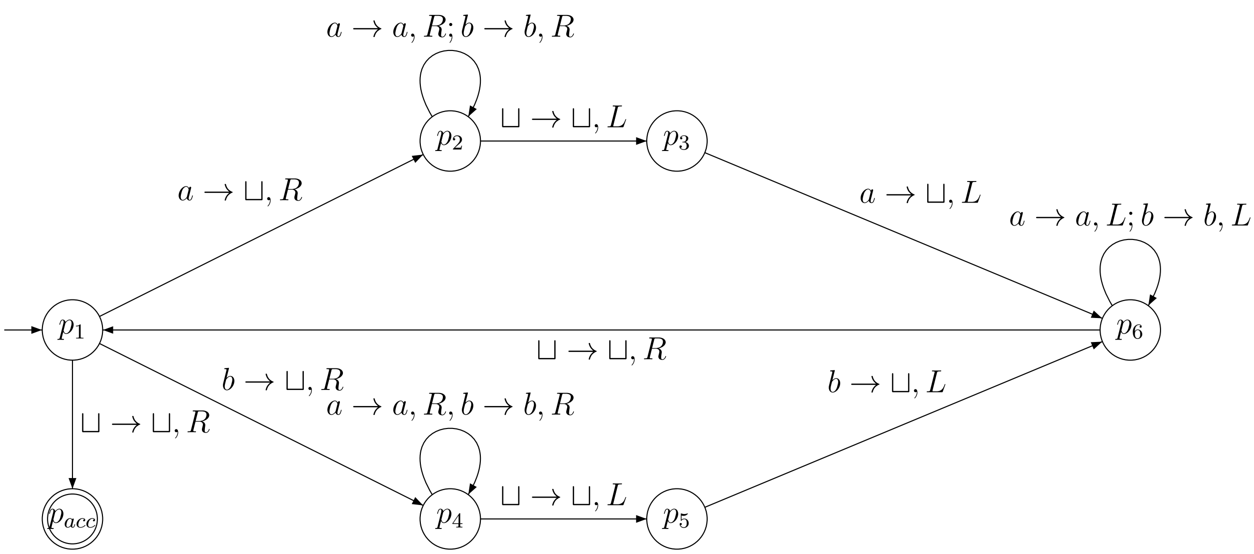
Check on input b, divide the first element into two sets:

$\{\{q1, q2\}, \{q5\}, \{q3, q4\}, \{q6\}\}$.

Since on both input a and b, the transitions lead us to the same destination set if we start from states in the same set. This gives us the following minimal DFA:



2. [**Category**: Turing Machine Comprehension, **Points**: 20]



Consider the above Turing machine M over the input alphabet $\Sigma = \{a, b\}$. The reject state p_{rej} is not shown, and all "missing" transitions are assumed to go to p_{rej} in M

(a) Give the formal tuple notation for M (Don't forget to specify the reject state in the tuple, but you don't need to specify it for delta function).

(b) Describe the computation of M on input *abba* as a sequence of configurations.

(c) What language does M recognize? Give an informal justification.

Solution:

(a) $M = \{Q, \Sigma, \Gamma, \delta, p_1, p_{acc}, p_{rej}\}$, where
$Q = \{p_1, p_2, p_3, p_4, p_5, p_6, p_{acc}, p_{rej}\}$,
$\Sigma = \{a, b\}$,
$\Gamma = \{a, b, \sqcup\}$, and δ is as follows:
$\delta(p_1, \sqcup) = (p_{acc}, \sqcup, R),\ \delta(p_1, a) = (p_2, \sqcup, R),\ \delta(p_1, b) = (p_4, \sqcup, R)$
$\delta(p_2, a) = (p_2, a, R),\ \delta(p_2, b) = (p_2, b, R),\ \delta(p_2, \sqcup) = (p_3, \sqcup, L)$
$\delta(p_3, a) = (p_2, \sqcup, L)$
$\delta(p_4, a) = (p_4, a, R),\ \delta(p_4, b) = (p_4, b, R),\ \delta(p_4, \sqcup) = (p_5, \sqcup, L)$
$\delta(p_5, b) = (p_5, \sqcup, L)$
$\delta(p_6, a) = (p_6, a, L),\ \delta(p_6, b) = (p_6, b, L),\ \delta(p_6, \sqcup) = (p_1, \sqcup, R)$

(b) $p_1abba \to \sqcup p_2bba \to \sqcup bp_2ba \to \sqcup bbp_2a \to \sqcup bbap_2\sqcup \to \sqcup bbp_3a\sqcup$
$\to \sqcup bp_6b\sqcup \to \sqcup p_6bb\sqcup \to p_6 \sqcup bb\sqcup \to \sqcup p_1bb\sqcup \to \sqcup \sqcup p_4b\sqcup \to \sqcup \sqcup bp_4\sqcup$
$\to \sqcup \sqcup p_5b\sqcup \to \sqcup p_6 \sqcup \sqcup\sqcup \to \sqcup \sqcup p_1 \sqcup \sqcup \to \sqcup \sqcup \sqcup p_{acc}\sqcup$

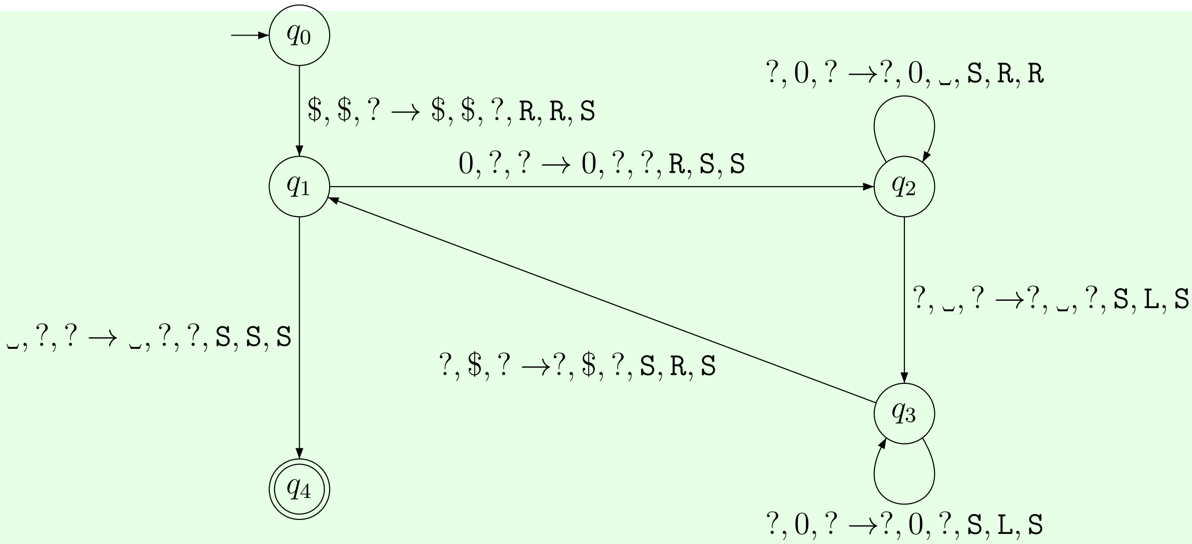
(c) $L(M) = \{ww^R | w \in \{a, b\}^*\}$. M markes off first letter then moves to the end of the letter to check whether the last letter is the same. If it's the same, then M markes it off, too. Then M moves the head to the first letter (of the substring, which is the original string without the first and the last letter), and performs the same process. So on and so forth. Once the string on the tape is empty (all $\sqcup$'s), M accepts the input string. Thus, the string must be the form of ww^R.

3. [**Category**: TM Design, **Points**: 20]

Design a *3-tape* TM for multiplication. The inputs are \$$0^a$ on ✇$_1$, \$$0^b$ on tape ✇$_2$, and blank on tape ✇$_3$. The TM should keep ✇$_1$ and ✇$_2$ unchanged, and write 0^{ab} on ✇$_3$.

Draw the TM using nodes and transitions, and describe the basic idea of your design. You don't need to give the formal description of your TM. You can also skip the reject state q_{rej}. All "missing" transitions in your TM are assumed to go to q_{rej} as per our convention.

Solution:



The idea is to read $\circledast_1$ once, from left to right. Whenever we read a 0 from $\circledast_1$, copy b 0's from $\circledast_2$ to $\circledast_3$. Conceptually, every time the loop starts from by reading a 0 from $\circledast_1$ (state q_1), followed by copying b 0's to $\circledast_3$ (state q_2), then move the head of $\circledast_2$ back to the beginning of the tape (state q_3). The loop terminates when ␣ is read from $\circledast_1$.

4. [**Category**: Encoding machines, **Points**: 20]

Give an encoding of DFAs over the alphabet $\{a, b\}$ using a string over the alphabet $\{a, b, \$\} \cup \{, \}$.

Using this give an encoding of a pair (D, w) where D is a DFA over $\{a, b\}$ and $w \in \{a, b\}^*$, and show that there is a TM that simulates accepts such an encoded pair iff D accepts w.

(Note that your TM has input alphabet $\Sigma = \{a, b, \$\} \cup \{, \}$.)

You may use a second tape if you like.