Problem Set 5
Fall 11

Due: Tuesday, 1st November, 2011, 11:00 am before class begins

Please <u>follow</u> the homework format guidelines posted on the class web page:
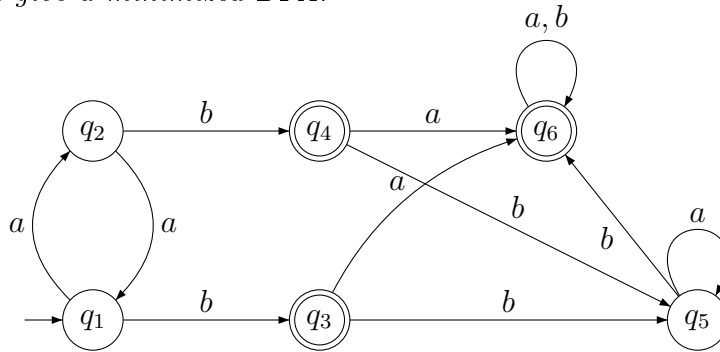
http://www.cs.illinois.edu/class/fa11/cs373/

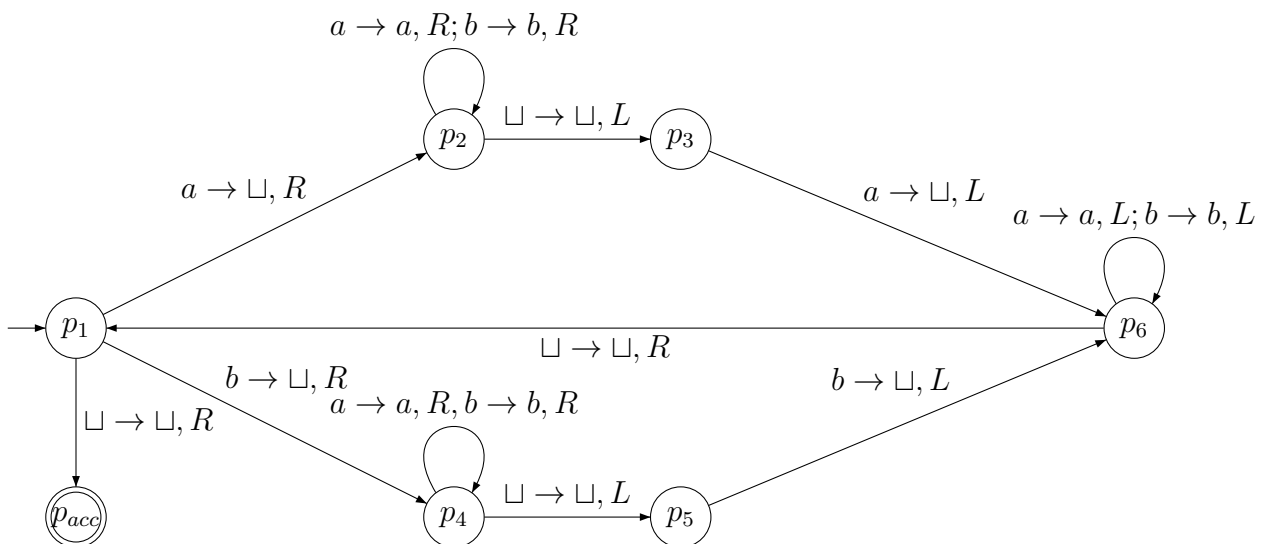1. [**Category**: DFA minimization, **Points**: 20]

    Minimize the above DFA using the partition refinement technique (see lecture notes).
    You must show the partitions in each step and the final minimized DFA as a diagram.
    Do not just give a minimized DFA.

2. [**Category**: Turing Machine Comprehension, **Points**: 20]

    Consider the above Turing machine M over the input alphabet $\Sigma = \{a, b\}$. The reject
    state p_{rej} is not shown, and all "missing" transitions are assumed to go to p_{rej} in M

(a) Give the formal tuple notation for M (Don't forget to specify the reject state in the tuple, but you don't need to specify it for delta function).

(b) Describe the computation of M on input *abba* as a sequence of configurations.

(c) What language does M recognize? Give an informal justification.

3. [**Category**: TM Design, **Points**: 20]

Design a *3-tape* TM for multiplication. The inputs are $\$0^a$ on ⊛$_1$, $\$0^b$ on tape ⊛$_2$, and blank on tape ⊛$_3$. The TM should keep ⊛$_1$ and ⊛$_2$ unchanged, and write 0^{ab} on ⊛$_3$.

Draw the TM using nodes and transitions, and describe the basic idea of your design. You don't need to give the formal description of your TM. You can also skip the reject state q_{rej}. All "missing" transitions in your TM are assumed to go to q_{rej} as per our convention.

4. [**Category**: Encoding machines, **Points**: 20]

Give an encoding of DFAs over the alphabet $\{a, b\}$ using a string over the alphabet $\{a, b, \$\} \cup \{, \}$.

Using this give an encoding of a pair (D, w) where D is a DFA over $\{a, b\}$ and $w \in \{a, b\}^*$, and show that there is a TM that simulates accepts such an encoded pair iff D accepts w.

(Note that your TM has input alphabet $\Sigma = \{a, b, \$\} \cup \{, \}$.)

You may use a second tape if you like.