Problem Set 4

Fall 11

Due: Thursday, 20th October, 2011, 11:00 am before class begins
Please follow the homework format guidelines posted on the class web page:
`http://www.cs.illinois.edu/class/fa11/cs373/`
Also, note that Problem 5 is an extra credit question.

1. [**Category**: Non-regularity, **Points**: 30]

   Prove the following languages are not regular, from first principles, using the Myhill-Nerode Theorem or using the pumping lemma.

   You cannot assume the non-regularity of *any* language to solve this problem.

   (a) $L = \{w \in \{a,b\}^* \mid N_b(w) = 2N_a(w)\}$ is non-regular. ($N_a(w)$ denotes the number of a's in w; hence L contains all words over $\{a,b\}$ in which the number of a's is precisely twice the number of b's.)

   (b) $L = \{0^{i^2+1} | i \geq 0\}$

   (c) Consider the following language L over the alphabet $\{0,1\}$:

   $$L = \{wtw \mid w, t \in \{0,1\}^+\}$$

   (Recall, $\{0,1\}^+$ is the set of all binary strings not including ϵ; i.e. $\{0,1\}^+ = \{0,1\}.\{0,1\}^*$.) Thus, $111011 \in L$, because $111011 = 11.10.11$. On the other hand, $10100 \notin L$ because there is no length $i > 0$ such that the first i symbols are the same as the last i symbols.

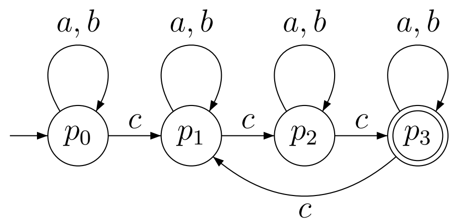
2. [**Category**: Non-regularity using closure properties, **Points**: 20]

   Prove that the following languages are not regular using *only* closure properties (you cannot use the Pumping Lemma or the MNT technique). You may assume that regular languages are closed under union, intersection, concatenation, Kleene-*, complement, and reverse. The only language you can assume non-regular is $\{0^n 1^n \mid n \geq 0\}$.

   (a) $L_{eq} = \{w \in \{0,1\}^* \mid N_0(w) = N_1(w)\}$ ($N_0(w)$ denotes the number of 0's in w and $N_1(w)$ denotes the number of 1's in w)

   (b) $L_2 = \{0^{2n} 1^{2n} \mid n \geq 0\}$

3. [**Category**: Suffix Language, **Points**: 20]

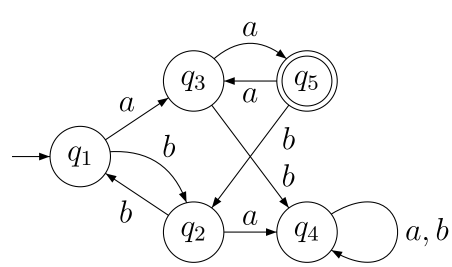


(a) For each state q in the above DFA, give the language accepted from q (i.e. L_q). Also, for each state q, give a string x such that $L_q = [\![L/x]\!]$.

(b) Prove that all the languages defined by the states are different: for each pair of states q, q', show that $L_q \neq L_{q'}$, by giving a string that belongs to one language but not the other. Note that you need to give 6 such strings, one for each pair of languages.

(c) Is this DFA a minimal DFA for the language it accepts? Why?

4. [**Category**: DFA Minimization, **Points**: 20]



Minimize the above DFA using the partition refinement technique (see lecture notes). Illustrate the partition at every step clearly. *Do not just give a minimized DFA.*

5. [**Category**: Extra Credit, **Points**: 20]

Let $\Sigma = \{a, b\}$ and let $L \subseteq \Sigma^*$ be a regular language.

Let L' be the set of words w such that changing one letter in w (from a to b or b to a) results in a word in L. I.e. L is the language of words that's exactly different from some word in L on one letter.

More precisely, $L' = \{waw' \mid wbw' \in L\} \cup \{wbw' \mid waw' \in L\}$.

Prove that L' is regular.