Problem Set 2

Fall 11

Due: 27th September, 2011, 11:00 am before class begins

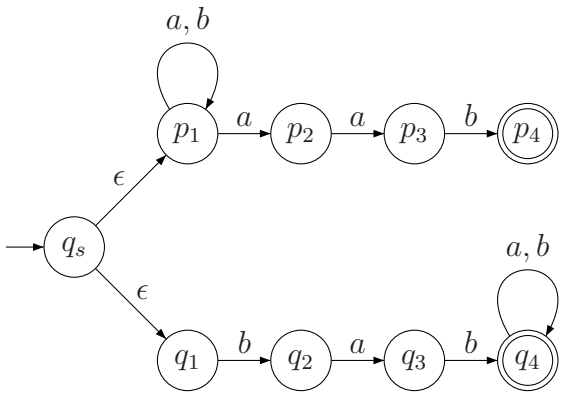
Please follow the homework format guidelines posted on the class web page:

`http://www.cs.illinois.edu/class/fa11/cs373/`

Also, note that Problem 6 is an extra credit question.

1. [**Category**: NFA Comprehension, **Points**: 20]

   Consider the following NFA M.



   (a) Formally show that M accepts the string $w_1 = abaaab$ and string $w_2 = babaab$.

   (b) Give a formal definition of the language that M recognizes. Briefly describe why M recognizes it.

2. [**Category**: NFA Construction, **Points**: 20]

   Construct a non-deterministic finite automata that accepts the language $\{01, 012\}^*$ over the alphabet, $\{0, 1, 2\}$. Your automata should contain only three states.

   Hint: Think nondeterminism, and ϵ is your friend.

3. [**Category**: Construction, **Points**: 20]

   For a string w, the reverse of w is defined as the string obtained by reading s from right to left, denoted by w^{-1}. For example, if $w = abc$, then $w^{-1} = cba$; if $w = abab$, then $w^{-1} = baba$.

   For a language L, the reverse of L is defined as the language

$$reverse(L) = \{w^{-1} \mid w \in L\}$$

Let $A = \{Q, \Sigma, \delta, q_0, F\}$ be a DFA accepting L, construct an NFA B with no more than $|Q| + 1$ states that will accept $reverse(L)$. Give the formal definition of B (in tuple notation, no diagram). You should also argue how/why this NFA works (intuitive explanation is enough).

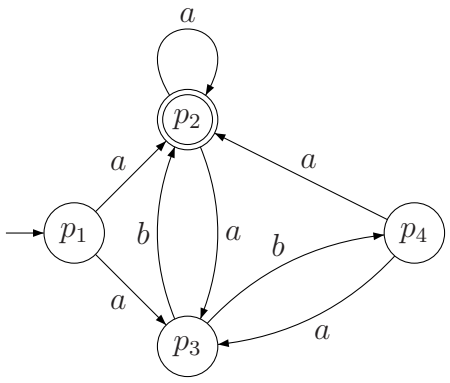
4. [**Category**: Regular Expressions, **Points**: 4+4+4+8]

   Give a regular expression for each of the following languages; the alphabet is $\{a, b\}$.

   - The set of all words that end with a b.
   - The set of all words that begin with aa and end with ab.
   - The set of all words such that every occurrence of a is immediately followed by a b.
   - The set of all words such that the number of changes from a to b is the same as the number of changes from b to a when read left to right.
     (E.g., *aabbbabbbba* is in the language, as there are two places where a's change to b's and two places where b's change to a's; however, *aabbbab* is not in the language as a's change to b's twice, while b's change to a's only once).

5. [**Category**: NFA to DFA Conversion, **Points**: 20]

   Convert the following NFA to a DFA using the subset construction, and show the state diagram.



   You can check your answer (if you wish) by feeding a DFA to the website:

   `http://pub.ist.ac.at/automata_tutor/solve?pid=16`

   However, the site does not check if you are describing a DFA; also note that you will lose points if you do not follow the subset construction.

6. [**Category**: Extra Credit:, **Points**: 20]

   Give a language L over the alphabet $\Sigma = \{a, b\}$ such that any DFA accepting L requires at least 3 final states. Prove that the language L you give has this property.