BIOE 298, SECTIONS MFI & B

PRACTICE EXAM 2

You have 80 minutes to complete this exam.
You may use notes or printouts from the course website,
but **no electronic resources**.

Circle your final answer for each question.

Part I (20 points; 4 points each)

(1) What is the distance between the origin and the hyperplane $3x_1 + 2x_2 - x_3 = 0$?

(2) True or False. The union of any two convex sets is also convex. (Note: the union of two sets is the set of all points contained in either set.)

(3) A genetic counselor informs a patient that their odds of getting disease are 1 in 300. What is the probability the patient will get the disease?

(4) True or False. Every square matrix has a pseudoinverse.

(5) True or False. If your dataset contains either categorical inputs or a categorial response variable, you should use logistic regression rather than linear regression.

Part II (20 points)

You are given a set of inputs ($\mathbf{x}$) and responses ($\mathbf{y}$)

$\mathbf{x}$	$\mathbf{y}$
0	2.1
1	3.1
2	5.8
3	11.3

You hypothesize that these data fit one of two models: $y = \beta_0 + \beta_1 x$ or $y = \beta_0 e^x$. You want to fit the parameters using linear regression.

Construct a design matrix for each model using the data in the above table. (The design matrix is the matrix $\mathbf{X}$ in the linear system $\mathbf{y} = \mathbf{X}\beta + \epsilon$.)

You use `fitlm` to fit both models, with the following outputs.

```
Linear regression model:
    y ~ 1 + x + x^2

Estimated Coefficients:
                   Estimate      SE        tStat      pValue
                   --------    -------    -------    --------

    (Intercept)     2.155      0.23974     8.9889    0.070533
    x              -0.345        0.385    -0.8961     0.53485
    x^2             1.125      0.12298     9.1476    0.069319


Number of observations: 4, Error degrees of freedom: 1
Root Mean Squared Error: 0.246
R-squared: 0.999,  Adjusted R-Squared 0.996
F-statistic vs. constant model: 421, p-value = 0.0344
```

```
Linear regression model:
    y ~ 1 + expx

Estimated Coefficients:
                   Estimate       SE       tStat      pValue
                   --------    --------    ------    ---------

    (Intercept)     1.8553      0.25642    7.2353     0.018572
    expx           0.47699     0.023746    20.087    0.0024692


Number of observations: 4, Error degrees of freedom: 2
Root Mean Squared Error: 0.355
R-squared: 0.995,  Adjusted R-Squared 0.993
F-statistic vs. constant model: 403, p-value = 0.00247
```

Which model would give better predictions? Why?

Part III (30 points)

Convert the following linear program to standard form:

$$\max f = 2x_1 - x_2$$

subject to

$$x_1 + x_2 \geq 5$$
$$2x_1 - x_2 \leq 4$$
$$x_1,\, x_2 \geq 0$$

Set up a Simplex tableau for the problem.

Solve and report (a) the optimal objective value, and (b) the optimal solution.

Part IV (30 points)

Find the intersection of the planes defined by

$$\begin{aligned} 3x - 4y + 2z &= 10 \\ 2x - z &= -2 \\ -x - 4y + 4z &= 14 \end{aligned}$$

Give a geometric interpretation of the intersection of these planes.