Problems 20

$$\frac{dx_1}{dt} = x_1 + 2x_2 \qquad x_1(0) = 0$$

$$\frac{dx_2}{dt} = 3x_1 + 2x_2 \qquad x_2(0) = -4$$

1. $$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

2. Find eigenvalues/eigenvectors for $\underline{A} = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

$$eig(A) : \quad [V, D] = eig(A)$$

V: eigenvectors

D: diagonal matrix of eigenvalues.

$$\lambda_1 = -1, \quad \underline{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 4, \quad \underline{v}_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\underline{x}(t) = k_1 \underline{v}_1 e^{\lambda_1 t} + k_2 \underline{v}_2 e^{\lambda_2 t}$$

$$= k_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + k_2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{4t} \quad , \quad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

when $t = 0$

$$\begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} k_1 + \begin{pmatrix} 2 \\ 3 \end{pmatrix} k_2 = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

$$\left[\begin{array}{ccc} -1 & 2 & 0 \\ 1 & 3 & -4 \end{array}\right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc} -1 & 2 & 0 \\ 0 & 5 & -4 \end{array}\right]$$

$$x_2 = -4/5$$

$$-x_1 + 2(-4/5) = 0 \Rightarrow x_1 = -8/5$$

$$\underline{x}(t) = -\frac{8}{5} \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} - \frac{4}{5} \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{4t}$$

Convert higher order linear ODEs into linear systems?

$$3\frac{dx}{dt} + 4\frac{d^2x}{dt} - x = 0 \Rightarrow \frac{d^2x}{dt} = -\frac{3}{4}\frac{dx}{dt} + \frac{1}{4}x$$

$$y_1 = x$$
$$y_2 = \frac{dx}{dt} \longrightarrow \frac{dy_1}{dt} = \frac{dx}{dt} = y_2$$

$$\frac{dy_2}{dt} = \frac{d^2x}{dt} = -\frac{3}{4}\frac{dx}{dt} + \frac{1}{4}x$$

$$= -\frac{3}{4}y_2 + \frac{1}{4}y_1$$

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1/4 & -3/4 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$