PROBLEMS 18

Do the vectors

$$\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\end{pmatrix}, \begin{pmatrix}1\\2\\0\end{pmatrix} \text{ span } \mathbb{R}^3$$

$\forall\ \underline{u} \in \mathbb{R}^3$, does $\exists\ a_1, a_2, a_3$ s.t.

$$\underline{u} = a_1\begin{pmatrix}1\\0\\0\end{pmatrix} + a_2\begin{pmatrix}0\\2\\0\end{pmatrix} + a_3\begin{pmatrix}1\\2\\0\end{pmatrix} \ ?$$

<u>No</u>.

$$u_3 = a_1 0 + a_2 0 + a_3 0 = 0$$

If 3 vectors span $\mathbb{R}^3 \Rightarrow$ the vectors are a basis.

$\Rightarrow$ they are linearly ind.

$$\begin{pmatrix}1\\2\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\2\\0\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & 1\\0 & 2 & 2\\0 & 0 & 0\end{pmatrix} \quad \text{Rank} \le 2$$

Are $\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}-1\\3\end{pmatrix},$ + $\begin{pmatrix}5\\2\end{pmatrix}$ linearly independent in $\mathbb{R}^2$.

No.

The basis $\mathbb{R}^2$ have 2 vectors.

If I had 3 linearly independent vectors, they would be a basis.

In $\mathbb{R}^n$, there can be no set of m lin. ind. vectors if $m > n$.

If they are lin. dep. $\exists$ a_1, a_2 s.t.

$$\begin{pmatrix}5\\2\end{pmatrix} = a_1\begin{pmatrix}1\\2\end{pmatrix} + a_2\begin{pmatrix}-1\\3\end{pmatrix}$$

$$\begin{pmatrix}1 & -1\\2 & 3\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix}$$

$$\begin{pmatrix}5\\2\end{pmatrix} = \frac{17}{5}\begin{pmatrix}1\\2\end{pmatrix} - \frac{8}{5}\begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix}$$

$$\begin{pmatrix}1 & -1 & 5\\2 & 3 & 2\end{pmatrix} \xrightarrow{R_2-2R_1} \begin{pmatrix}1 & -1 & 5\\0 & 5 & -8\end{pmatrix} \xrightarrow{R_2/5} \begin{pmatrix}1 & -1 & 5\\0 & 1 & -8/5\end{pmatrix}$$

$a_2 = -8/5$

$a_1 = -8/5 + 5 = 17/5$

Show that $\begin{pmatrix}1\\1\end{pmatrix}$ & $\begin{pmatrix}-1\\1\end{pmatrix}$ span $\mathbb{R}^2$.

$$\underline{u} = \begin{pmatrix}u_1\\u_2\end{pmatrix} = a_1\begin{pmatrix}1\\1\end{pmatrix} + a_2\begin{pmatrix}-1\\1\end{pmatrix}$$

$$a_1 - a_2 = u_1$$
$$a_1 + a_2 = u_2$$

$$\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix} = \begin{pmatrix}u_1\\u_2\end{pmatrix}$$

$$\begin{pmatrix}1 & -1 & u_1\\1 & 1 & u_2\end{pmatrix}$$

$$\xrightarrow{R_2 - R_1}\begin{pmatrix}1 & -1 & u_1\\0 & 2 & u_2 - u_1\end{pmatrix}$$

$$\xrightarrow{R_2/2}\begin{pmatrix}1 & -1 & u_1\\0 & 1 & \frac{u_2 - u_1}{2}\end{pmatrix}$$

$$\xrightarrow{R_1 + R_2}\begin{pmatrix}1 & 0 & u_1 + \frac{u_2 - u_1}{2}\\0 & 1 & \frac{u_2 - u_1}{2}\end{pmatrix}$$

$$= \begin{pmatrix}1 & 0 & \frac{u_1}{2} + \frac{u_2}{2}\\0 & 1 & \frac{u_2 - u_1}{2}\end{pmatrix}$$

$$a_1 = \frac{u_1 + u_2}{2}, \quad a_2 = \frac{u_2 - u_1}{2}$$

$$\underline{u} = \begin{pmatrix}-2\\4\end{pmatrix} \Rightarrow a_1 = 1, \quad a_2 = 3$$

$$\begin{pmatrix}-2\\4\end{pmatrix} = 1\cdot\begin{pmatrix}1\\1\end{pmatrix} + 3\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}1-3\\1+3\end{pmatrix} = \begin{pmatrix}-2\\4\end{pmatrix}$$